cmpe 362- Signal Processing

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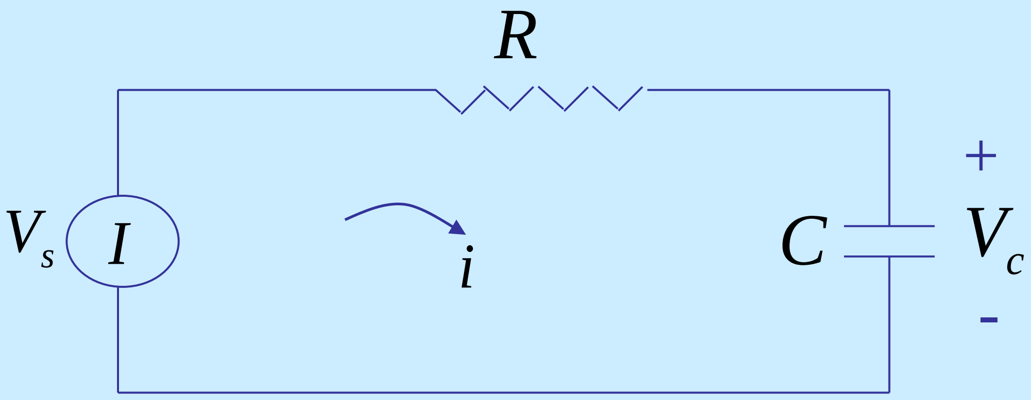
What is Signal?

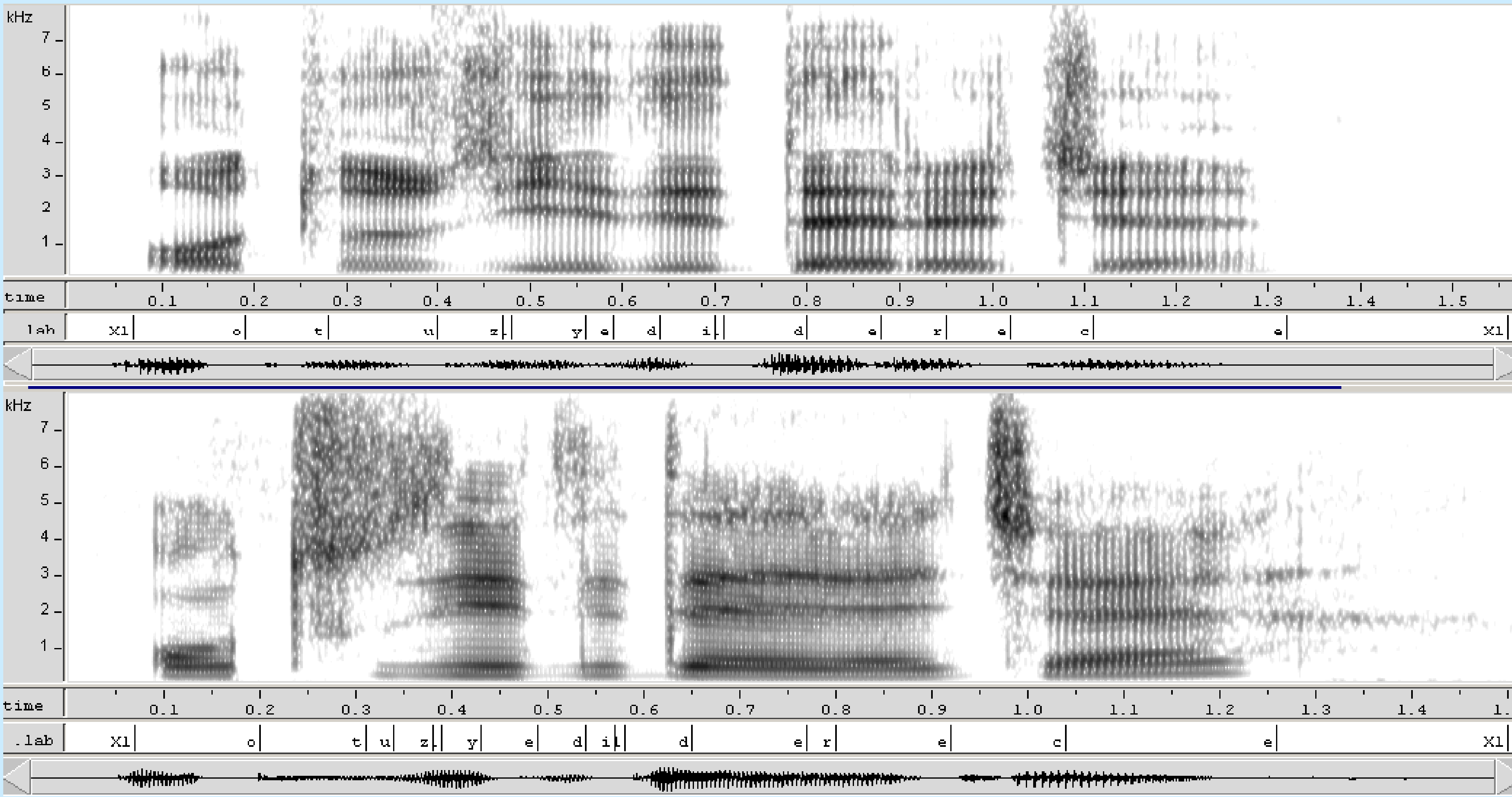
- **Signal** is the variation of a physical phenomenon / quantity with respect to one or more independent variable
- A signal is a function.

Example 1: Voltage on a capacitor as a function of time.



RC circuit

What is Signal?



Example 2: Two different vocoder signals of "Otuz yedi derece"
male/female, emotion, background noise, any aspects of vocoder functionallity. Have you had something spicy, salty, sour just before you say it; function of everything !!!

What is Signal?

Example 3 : Closing value of the stock exchange index as a function of days

Example 4:Image as a function of x-y coordinates (e.g. 256 X 256 pixel image)

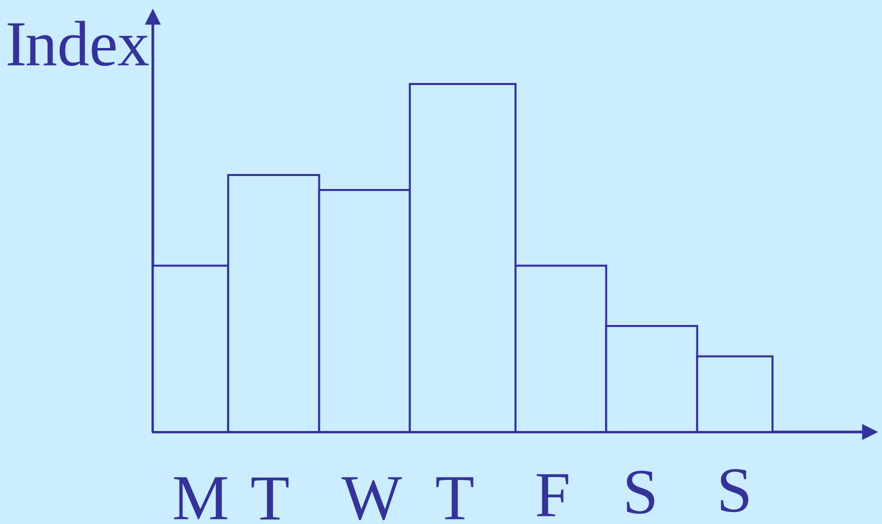
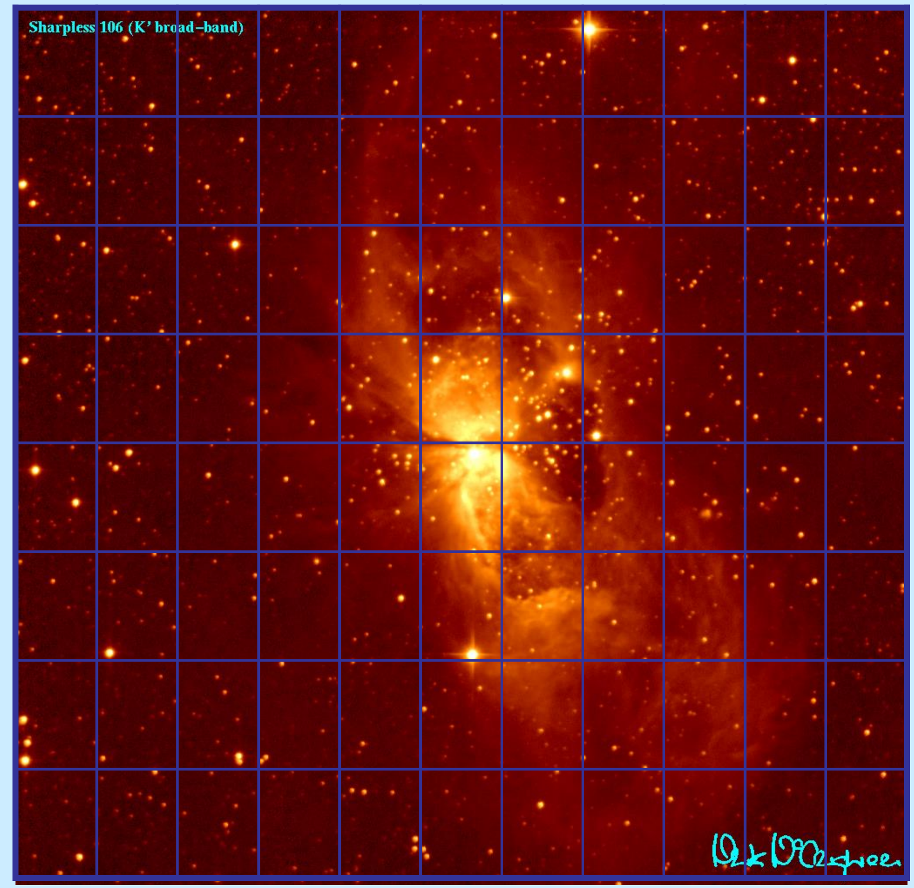


Fig. Stock exchange



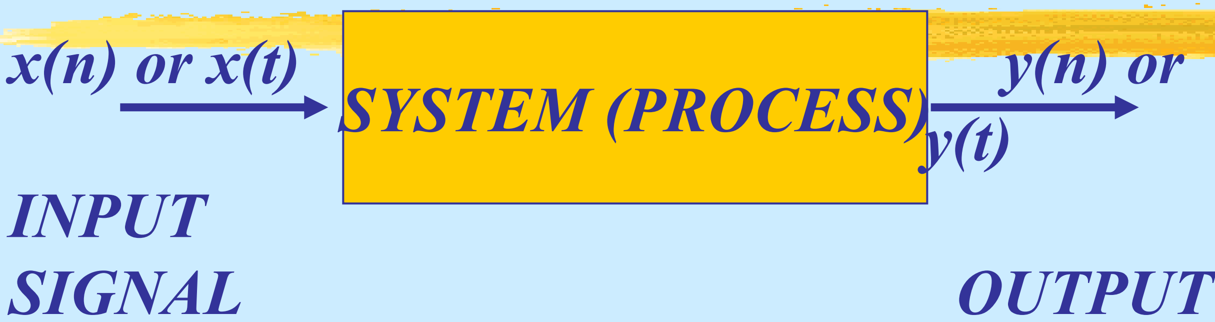
What is Signal Processing

- Process signal(s) for solving many scientific/ engineering/ theoretical purposes

- The process is includes calculus, Differential equations, Difference equations, Transform theory, Linear time-invariant system theory, System identification and classification, Time-frequency analysis, Spectral estimation, Vector spaces and Linear algebra, Functional analysis, statistical signals and stochastic processes, Detection theory, Estimation theory, Optimization, Numerical methods, Time series, Data mining, etc

- "The IEEE Transactions on Signal Processing covers novel theory, algorithms, performance analyses and applications of techniques for the processing, understanding, learning, retrieval, mining, and extraction of information from signals. The term "signal" includes, among others, audio, video, speech, image, communication, geophysical, sonar, radar, medical and musical signals. Examples of topics of interest include, but are not limited to, information processing and the theory and application of filtering, coding, transmitting, estimating, detecting, analyzing, recognizing, synthesizing, recording, and reproducing signals" REFERENCE: IEEE Transactions on Signal Processing Journal

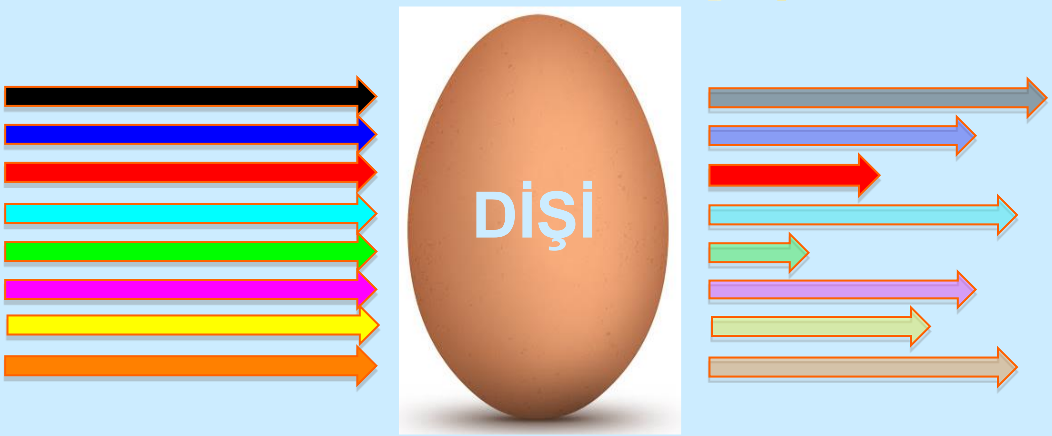
Signal Processing everywhere

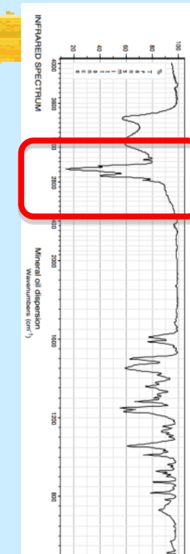


- Audio, speech, image, video
- Ranging from nanoscale to deepspace communications
- Sonar, radar, geosensing
- Array processing,
- Control systems (all industry)
- Seismology,
- Meteorology,
- Finance,
- Health, etc..

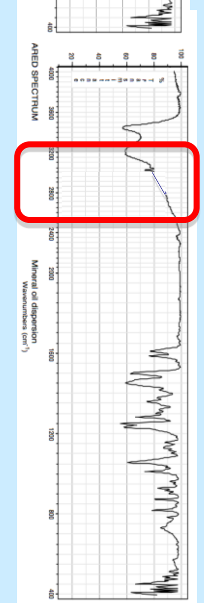
IT IS GOOD TO KNOW THIS COURSE

I am currently working on gender classification problem for chicken hatchery eggs





FTIR (Fourier Transform Infrared Spectrometer) yöntemi ile allantoik sıvının kimyasal bileşenine (östrojen seviyesi gibi) duyarlı frekans bandındaki (kırmızı bant) ışığın soğurulma oranına dayalı olarak yumurtadaki östrojen miktarı yumurtaya zarar vermeden (uzaktan) saptanabilir.



The course in a single slide

ALL ABOUT THREE DOMAINS

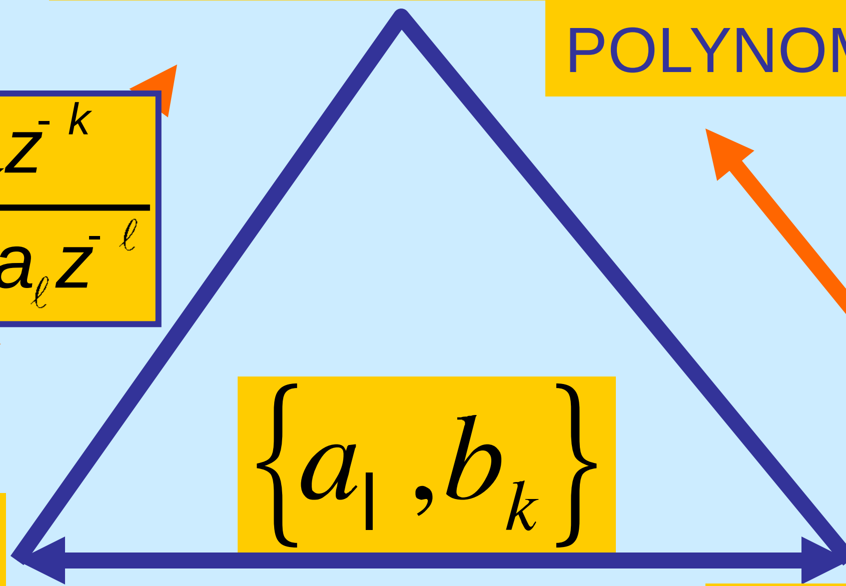
Use H(z) to get Freq. Response

$$H(z) = \frac{\sum b_k z^{-k}}{1 - \sum a_\ell z^{-\ell}}$$

Z-TRANSFORM-DOMAIN: poles & zeros

POLYNOMIALS: H(z)

$$z = e^{j\hat{w}}$$



$$\{a_\ell, b_k\}$$

TIME-DOMAIN

$$y[n] = \sum_{\ell=1}^{N} a_\ell y[n-\ell] + \sum_{k=0}^{M} b_k x[n-k]$$

FREQ-DOMAIN

$$H(e^{j\hat{\omega}}) = \frac{\sum_{k=0}^{M} b_k e^{-j\hat{\omega}k}}{1 - \sum_{\ell=1}^{N} a_\ell e^{-j\hat{\omega}\ell}}$$

Lets start from the beginning…

Signals…

Signal Processing First

LECTURE #1
Sinusoids

READING ASSIGNMENTS

- This Lecture:
  - Chapter 2, Sects. 2-1 to 2-5

- Appendix A: Complex Numbers
- Appendix B: MATLAB
- Chapter 1: Introduction

LECTURE OBJECTIVES

- Write general formula for a “sinusoidal” waveform, or signal
- From the formula, plot the sinusoid versus time

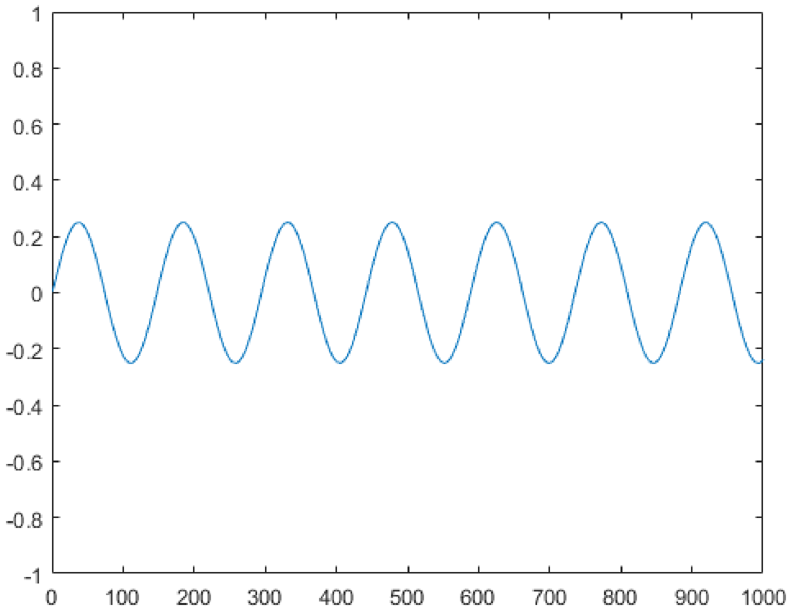
- What’s a **signal**?
  - It’s a **function** of time, x(t)
  - in the mathematical sense

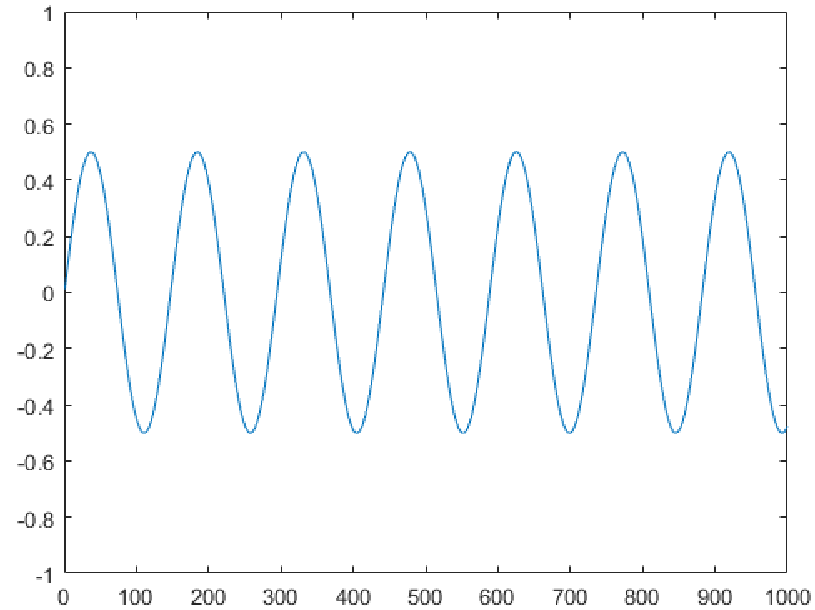
TUNING FORK EXAMPLE

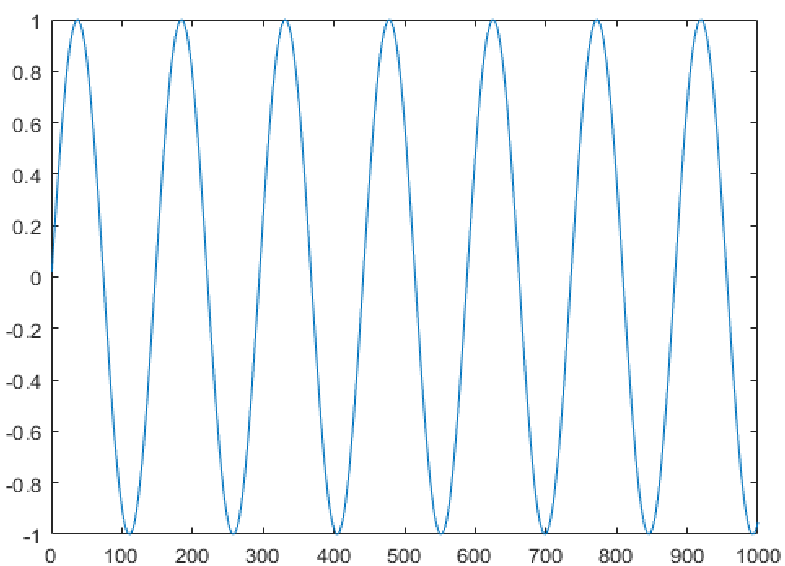
- CD-ROM demo
- "A" is at 440 Hertz (Hz)
- Waveform is a SINUSOIDAL SIGNAL
- Computer plot looks like a sine wave
- This should be the mathematical formula:

$$A\cos(2\pi(440)t + \varphi)$$

SINUSOID AMPLITUDE EXAMPLES







1-)0.25*sin(2π*300t)

2-)0.50*sin(2π*300t)

3-)1.00*sin(2π*300t)

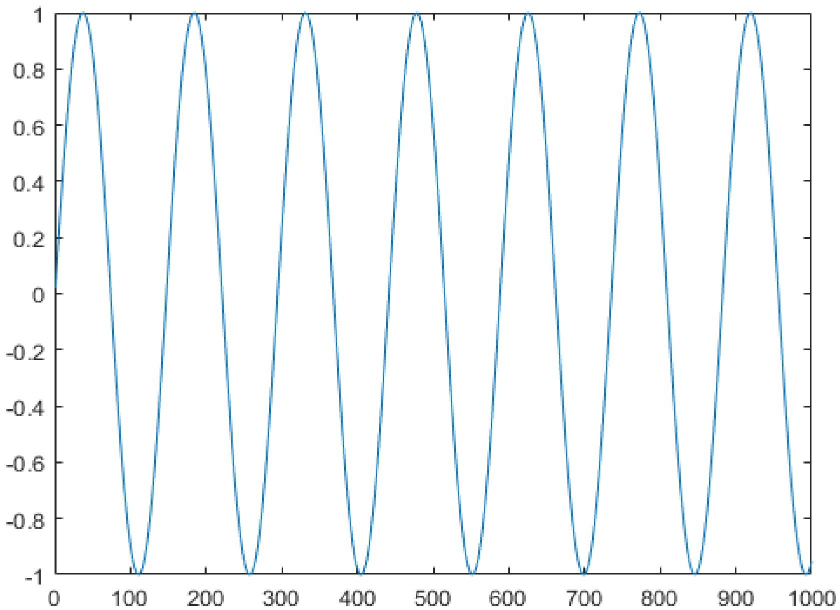
Different Amplitude Sound Examples

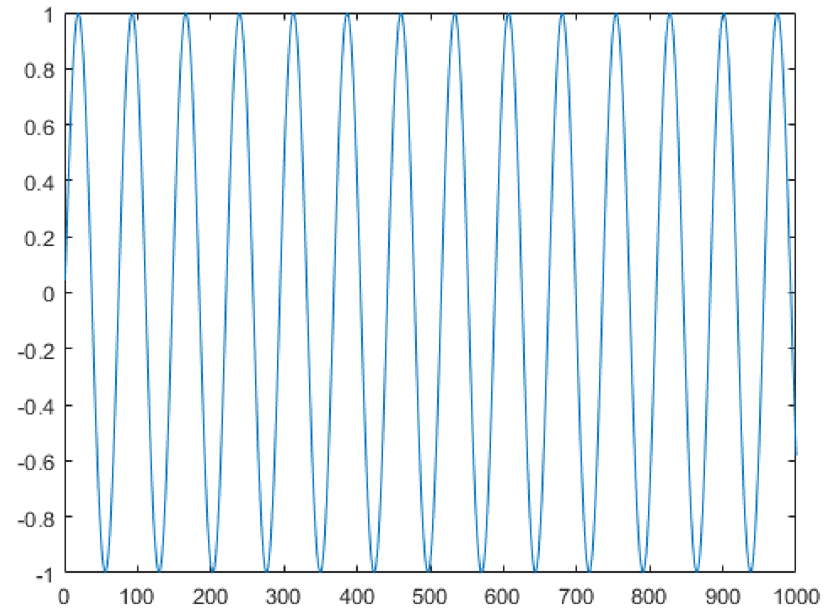
Amplitude = Asin(2π*300t)

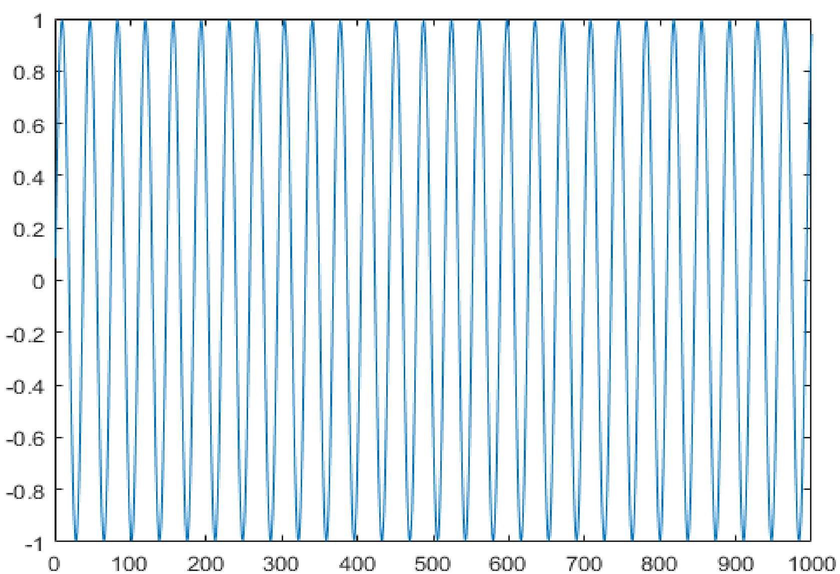
Amplitude = 5*Asin(2π*300t)

Amplitude =10*A sin(2π*300t)

SINUSOID FREQUENCY EXAMPLES







1-)1.00sin(2π*300t)
2-)1.00*sin(2π*600t)
3-)1.00*sin(2π*1200t)

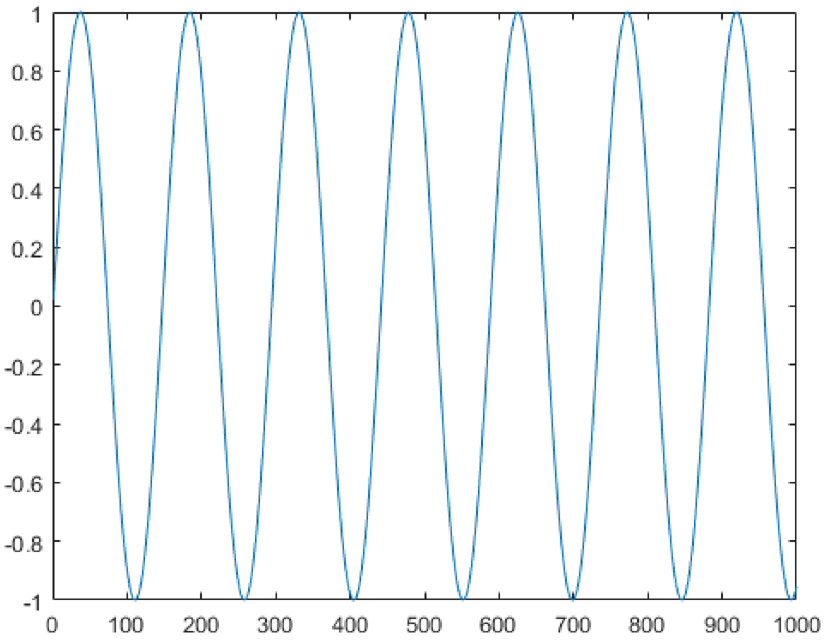
Different Amplitude Sound Examples

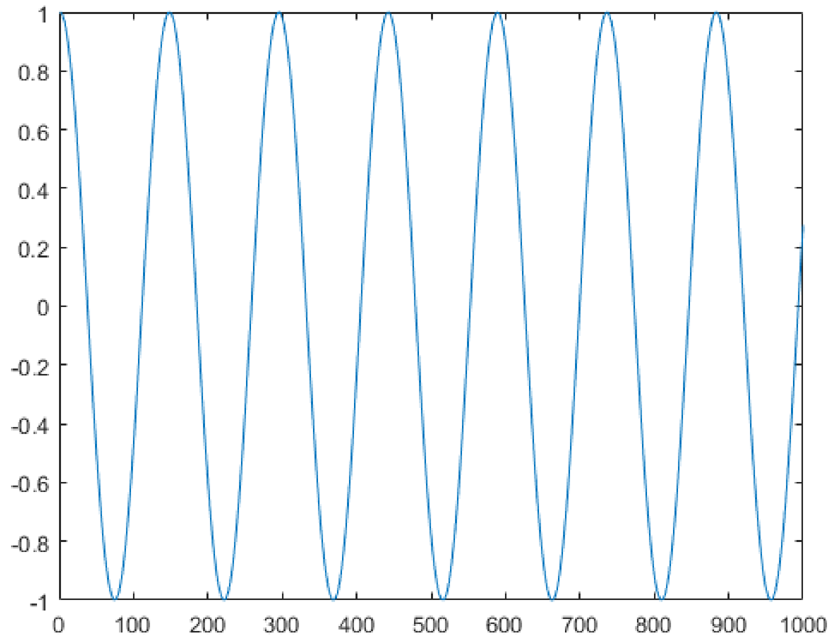
Amplitude = 1.00sin(2π*300t)

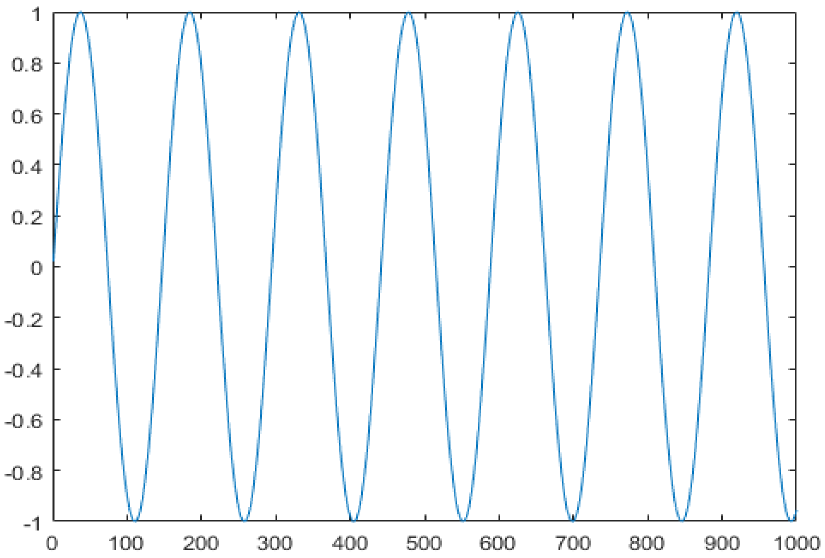
Amplitude = 1.00sin(2π*600t)

Amplitude = 1.00sin(2π*1200t)

SINUSOID PHASE EXAMPLES







1-)1.00sin(2π*300t)

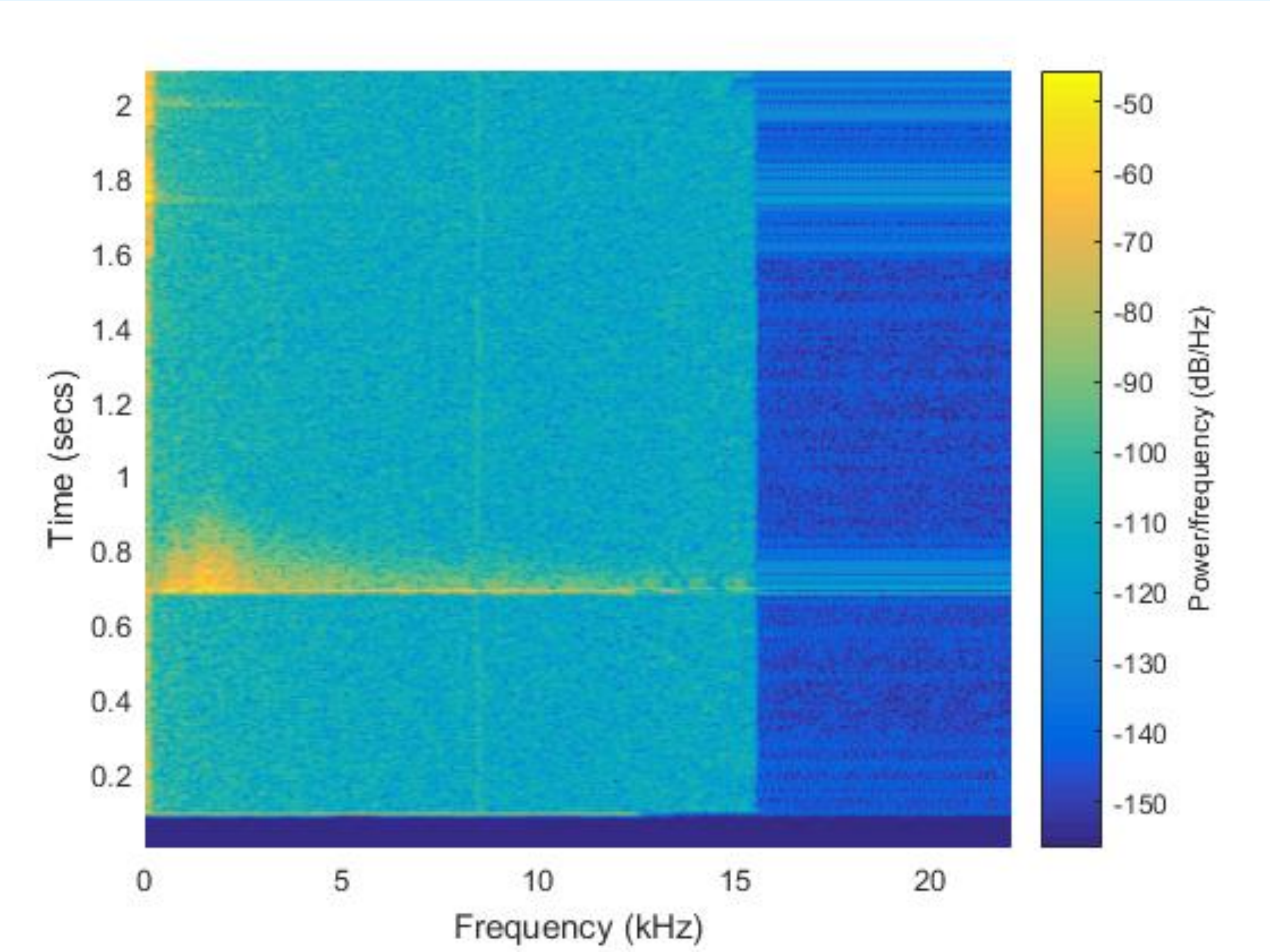
2-)1.00*sin(2π*300t+ π/2)

3-)1.00*sin(2π*300t+ π)

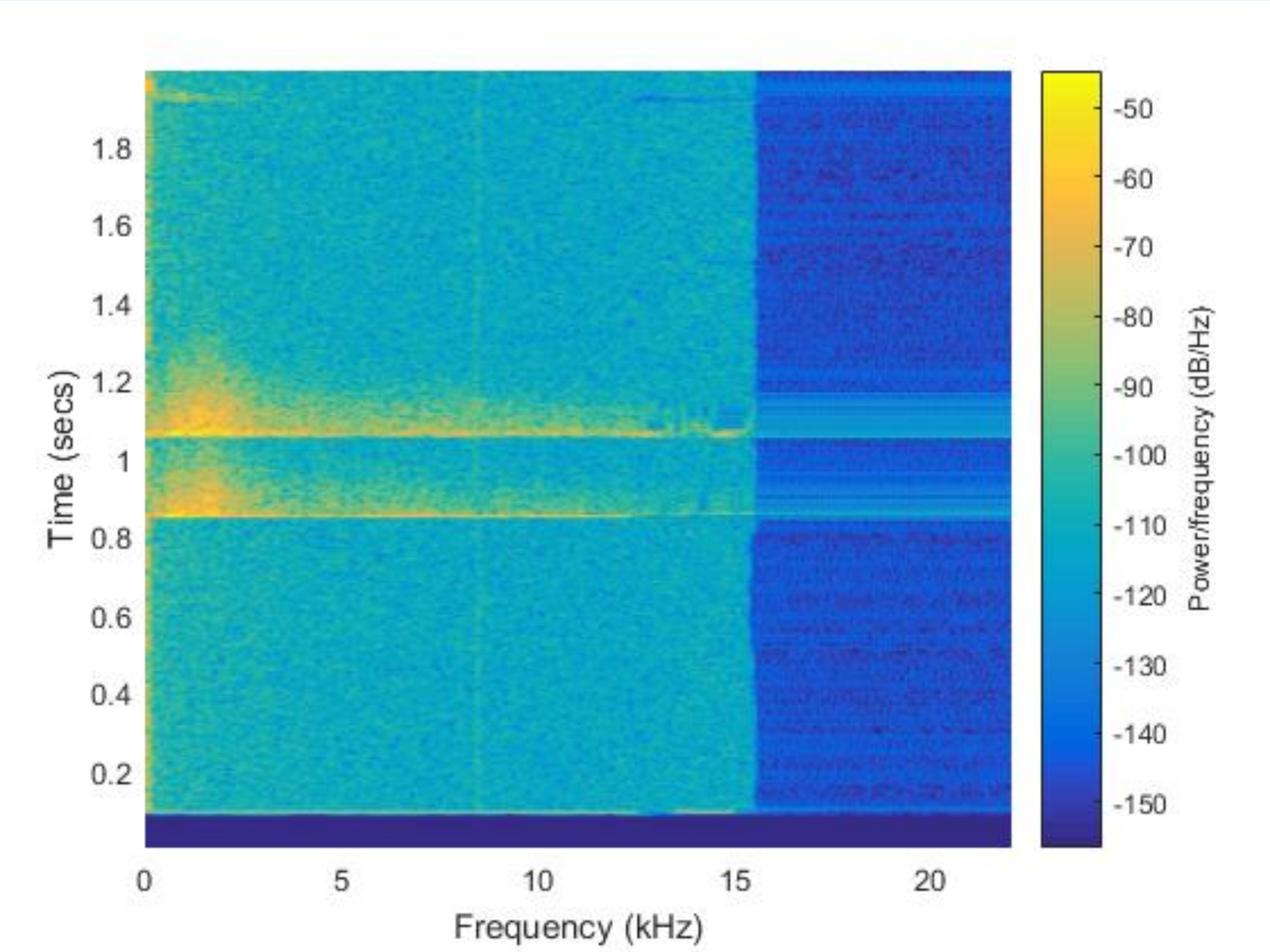
HUMAN VOICE WITH DIFFERENT SAMPLING FREQUENCIES

- With the original sampling frequency Fs
- When sampled with Fs/2
- When sampled with 2*Fs
- When sampled with 5*Fs

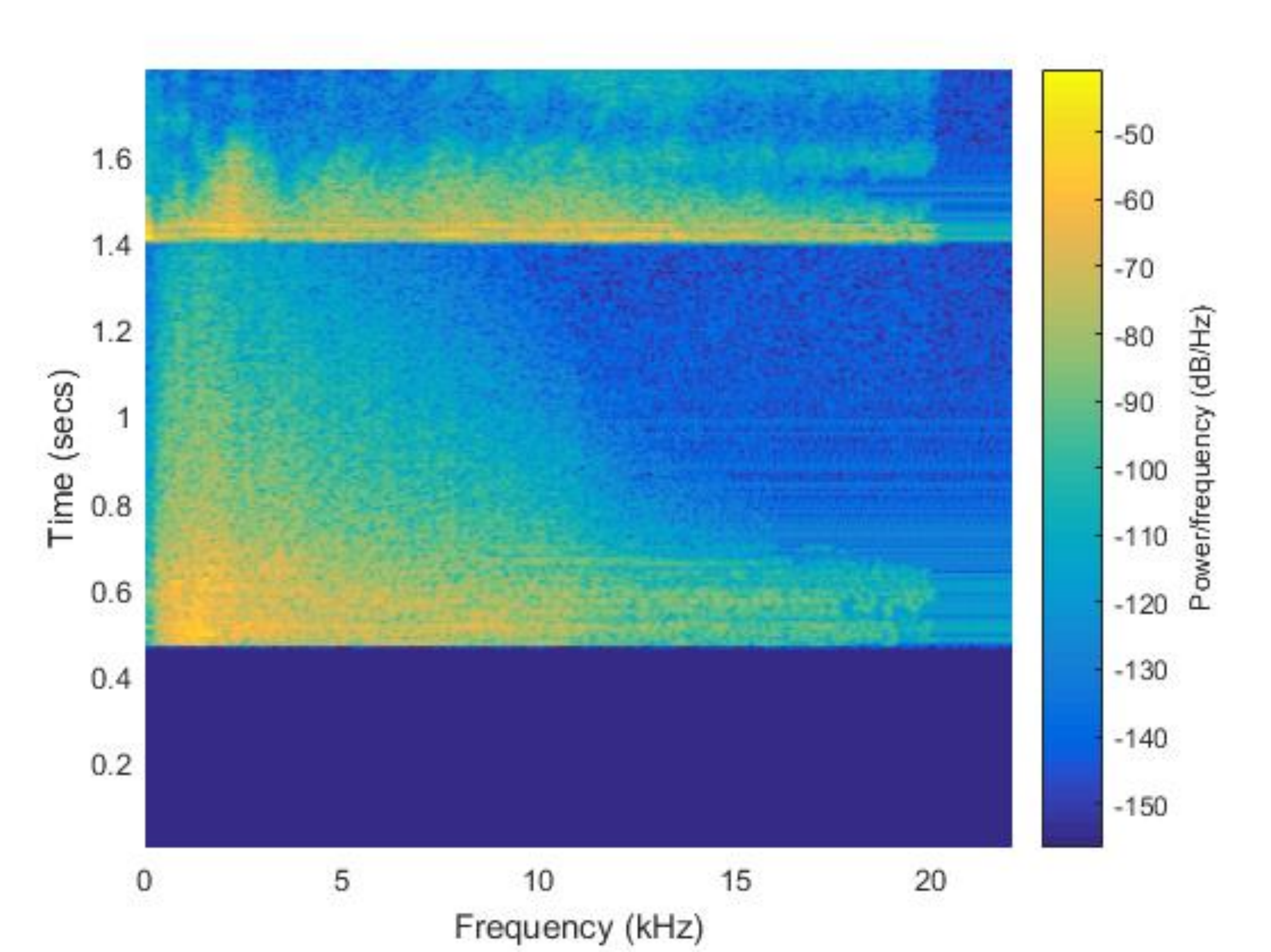
TIME VS FREQUENCY REPRESENTATION OF ONE CLAP RECORD(SPECTROGRAM)



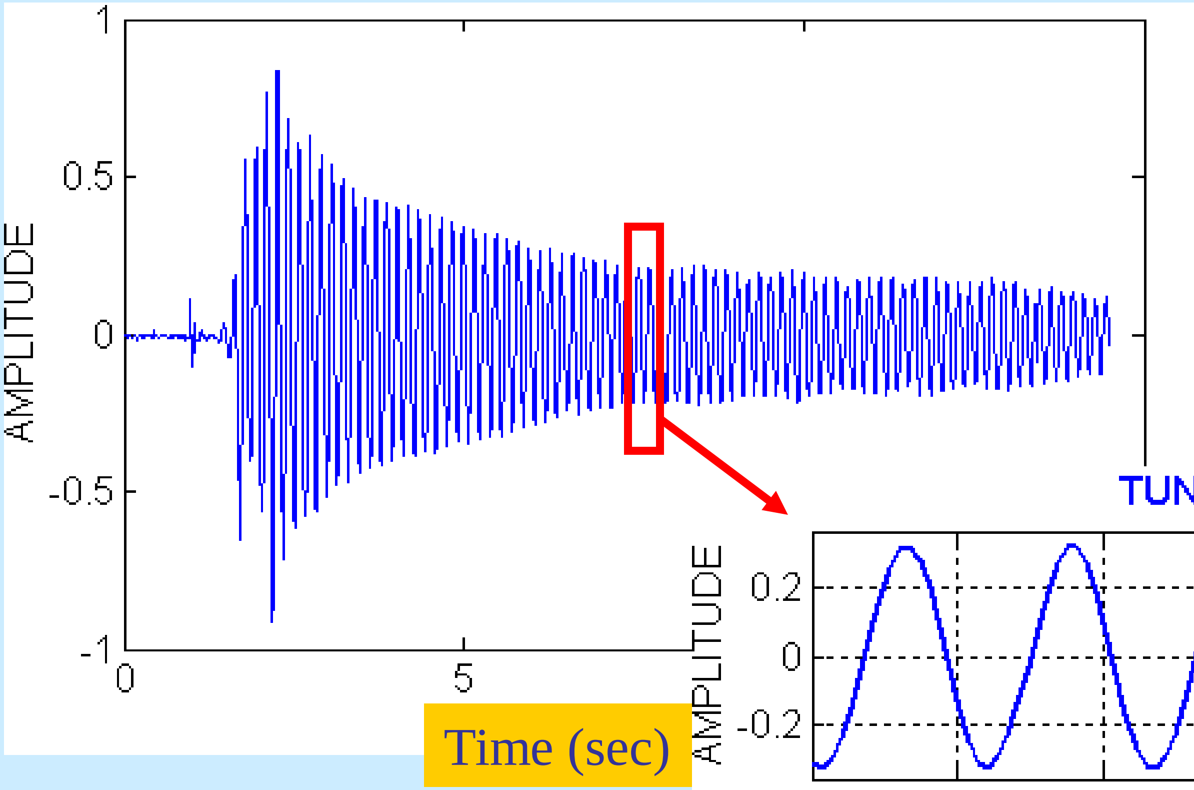
TIME VS FREQUENCY REPRESENTATION OF TWO CLAPS RECORD (SPECTROGRAM)



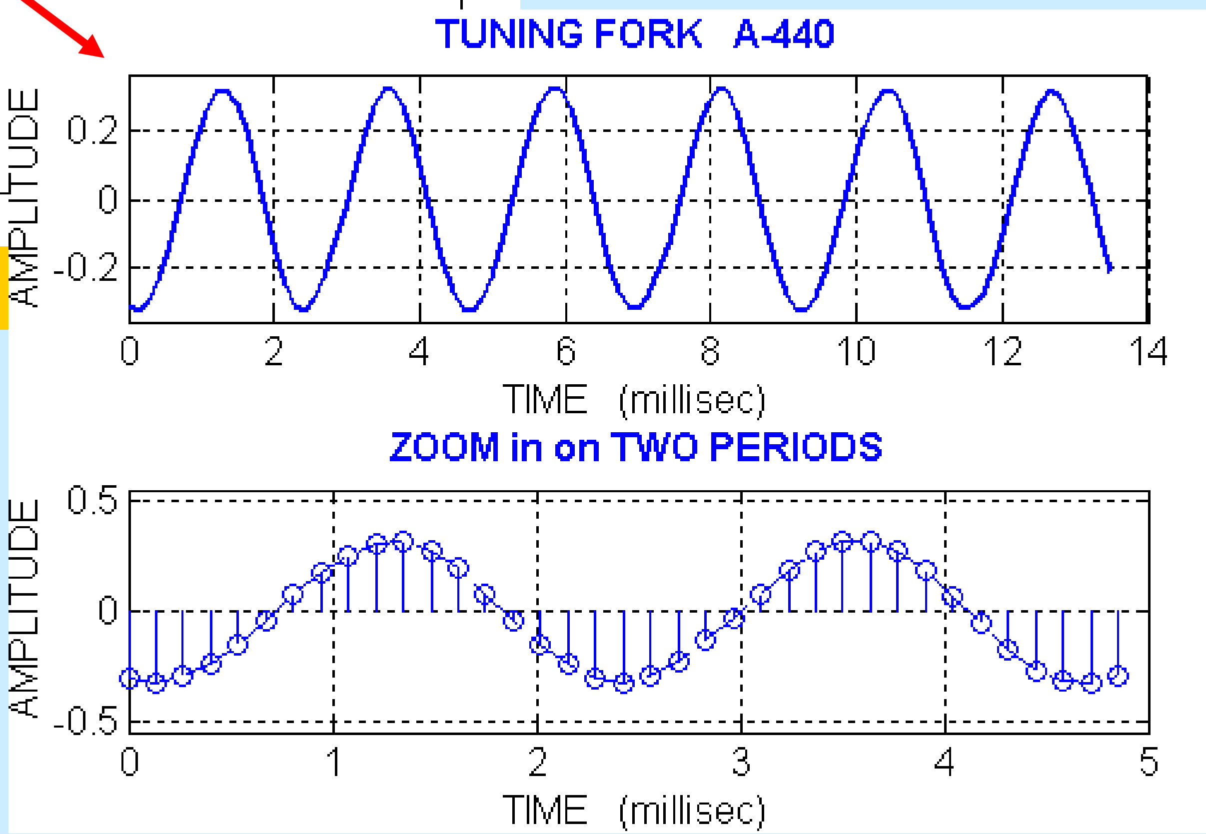
TIME VS FREQUENCY REPRESENTATION OF CLAP + SNAP RECORD (SPECTROGRAM) (FIRST CLAP THEN FINGER SNAP)



TUNING FORK A-440 Waveform



$$T \approx 8.15 - 5.85$$
$$= 2.3\ \text{ms}$$



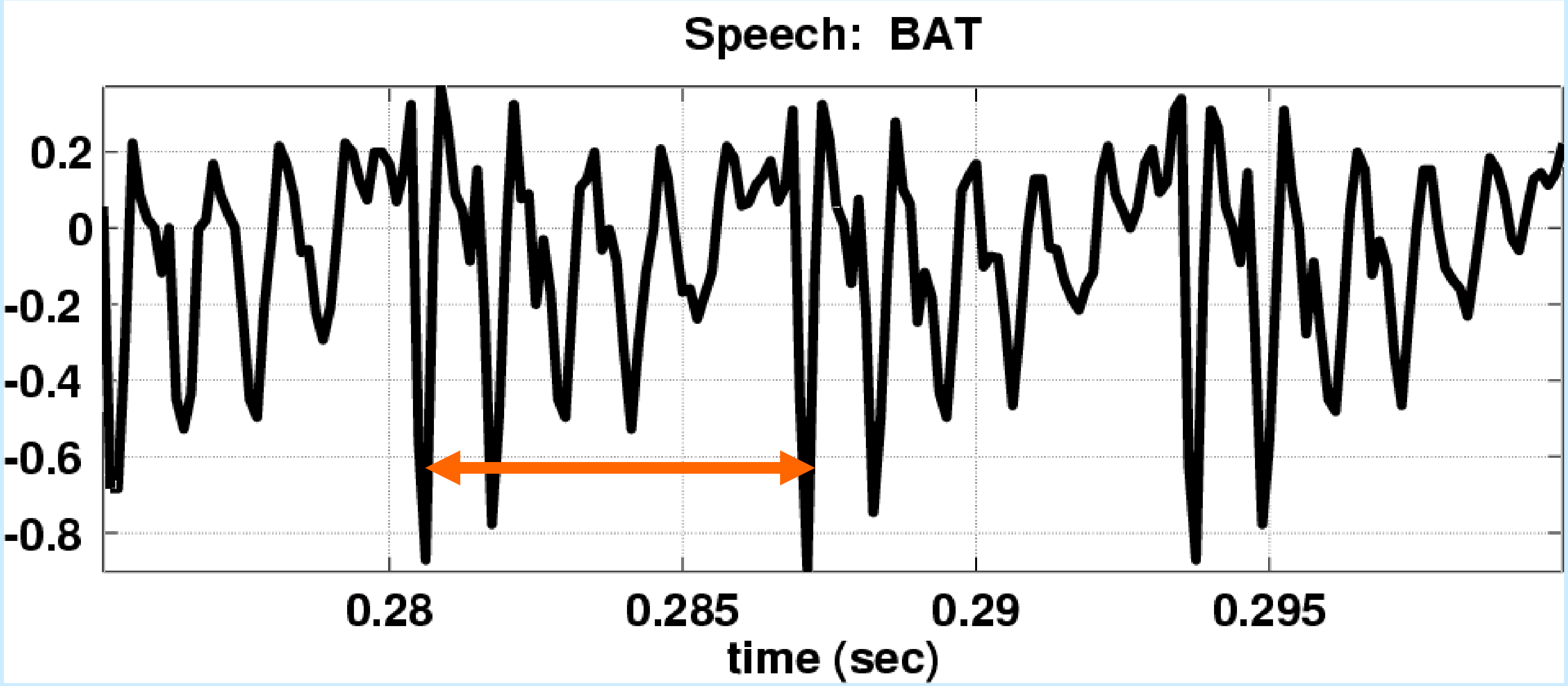
$$f = 1/T$$
$$= 1000/2.3$$
$$\approx 435\ \text{Hz}$$

SPEECH EXAMPLE

- More complicated signal (BAT.WAV)
- Waveform ***x*(*t*)** is NOT a Sinusoid
- Theory will tell us
  - ***x*(*t*)** is approximately a sum of sinusoids
  - FOURIER ANALYSIS
    - Break ***x*(*t*)** into its sinusoidal components
  - Called the FREQUENCY SPECTRUM

Speech Signal: BAT

- Nearly **Periodic** in Vowel Region
  - Period is (Approximately) T = 0.0065 sec



DIGITIZE the WAVEFORM

- ***x*[*n*]** is a SAMPLED SINUSOID
  - A list of numbers stored in memory
- Sample at 11,025 samples per second
  - Called the SAMPLING RATE of the A/D
  - Time between samples is
    - 1/11025 = 90.7 microsec
- Output via D/A hardware (at F_{samp})

STORING DIGITAL SOUND

- ***x*[*n*]** is a SAMPLED SINUSOID
  - A list of numbers stored in memory
- CD rate is 44,100 samples per second
- 16-bit samples
- Stereo uses 2 channels
- Number of bytes for 1 minute is
  - 2 X (16/8) X 60 X 44100 = 10.584 Mbytes

SINUSOIDAL SIGNAL

$$A\cos(wt + j\ )$$

- **FREQUENCY** w
  - Radians/sec
  - Hertz (cycles/sec)

$$w = (2p)f$$

- **PERIOD** (in sec)

$$T = \frac{1}{f} = \frac{2p}{w}$$

- **AMPLITUDE** A
  - Magnitude

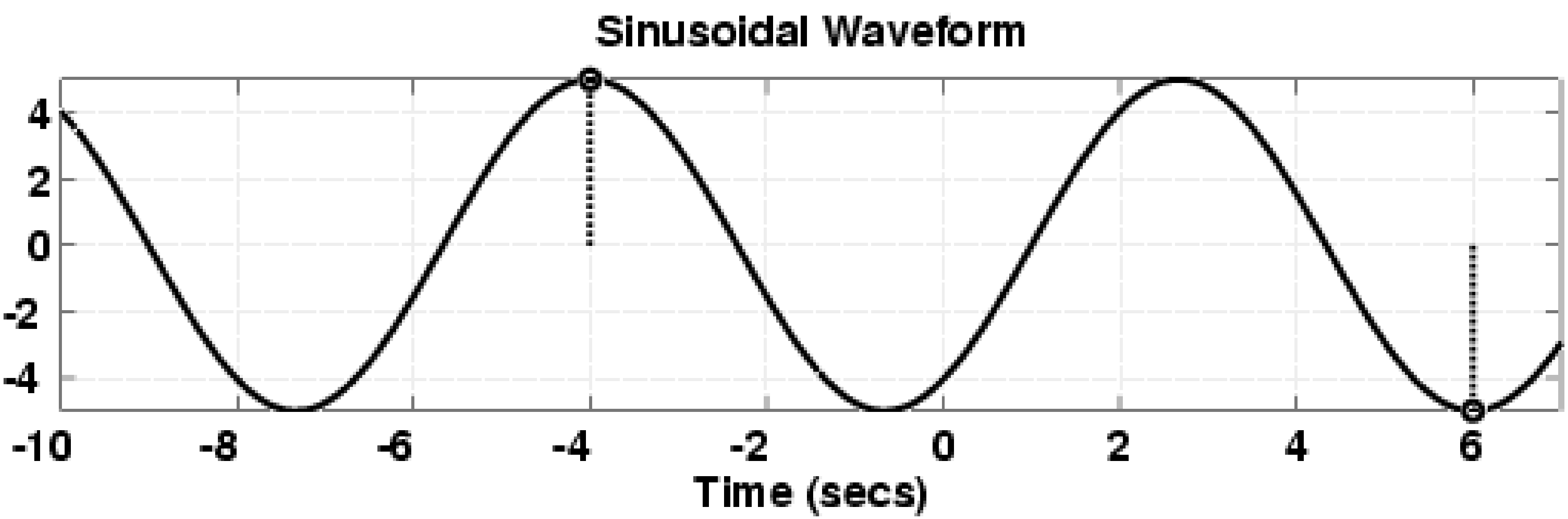
- **PHASE** j

EXAMPLE of SINUSOID

- Given the Formula

$$5\cos(0.3\pi t + 1.2\pi)$$

- Make a plot



PLOT COSINE SIGNAL

$$5\cos(0.3pt + 1.2p)$$

- Formula defines A, ω, and ϕ

$$A = 5$$

$$w = 0.3p$$

$$j = 1.2p$$

PLOTTING COSINE SIGNAL from the FORMULA

$$5\cos(0.3\pi t + 1.2\pi)$$

- Determine **period**:

$$T = 2\pi/\omega = 2\pi/0.3\pi = 20/3$$

- Determine a **peak** location by solving

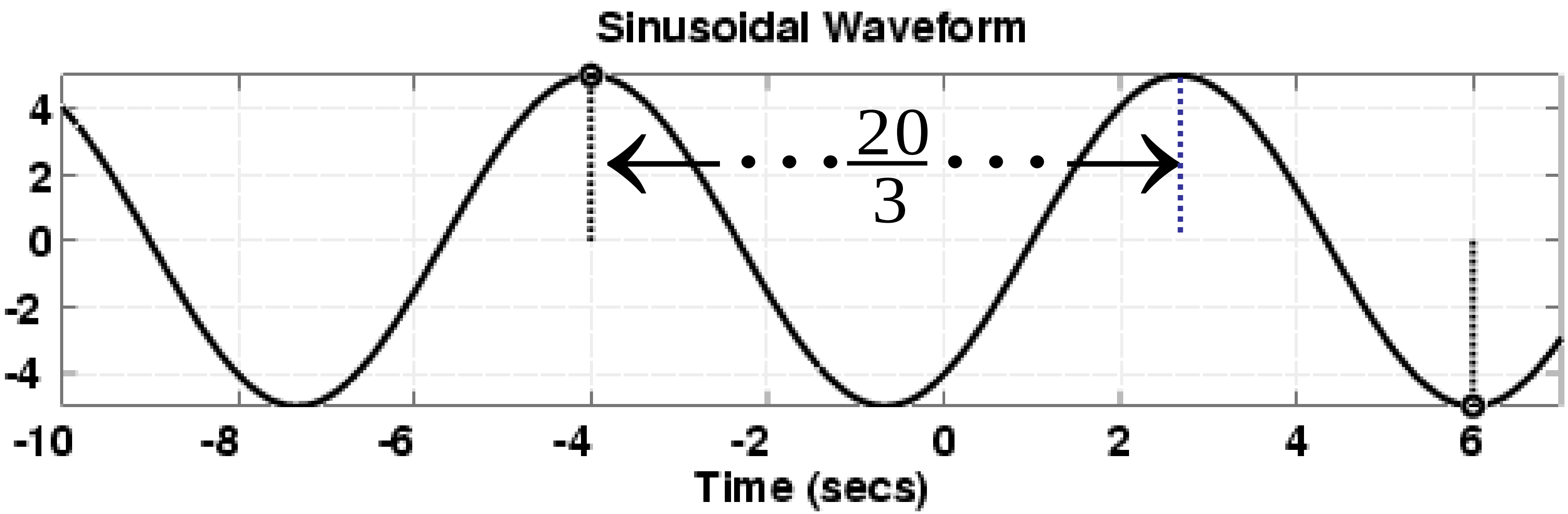
$$(\omega t + \varphi) = 0 \quad \Rightarrow (0.3\pi t + 1.2\pi) = 0$$

- **Zero** crossing is T/4 before or after
- Positive & Negative peaks spaced by T/2

PLOT the SINUSOID

$$5\cos(0.3\pi t + 1.2\pi)$$

- Use T=20/3 and the peak location at t=-4



PLOTTING COSINE SIGNAL from the FORMULA

$$5\cos(0.3\pi t + 1.2\pi)$$

- Determine **period**:

$$T = 2\pi/\omega = 2\pi/0.3\pi = 20/3$$

- Determine a **peak** location by solving

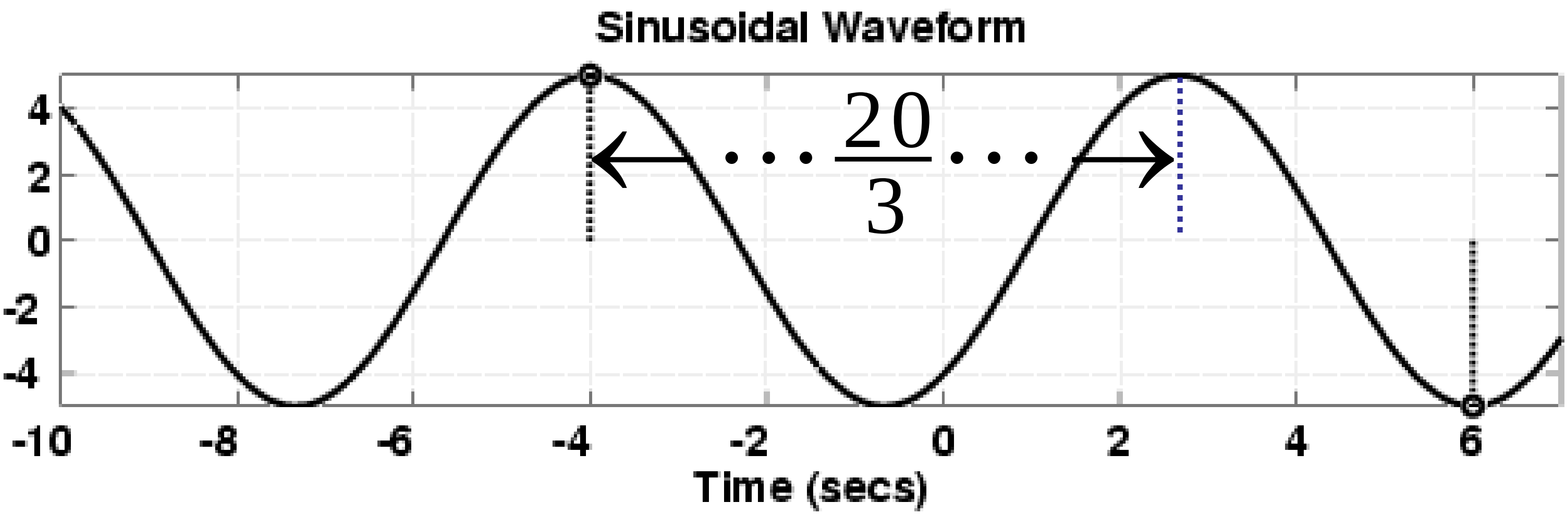
$$(\omega t + \varphi) = 0$$

- **Peak at t=-4**

ANSWER for the PLOT

$$5\cos(0.3\pi t + 1.2\pi)$$

- Use T=20/3 and the peak location at t=-4



TIME-SHIFT

- In a mathematical formula we can replace t with $t\text{-}t_m$

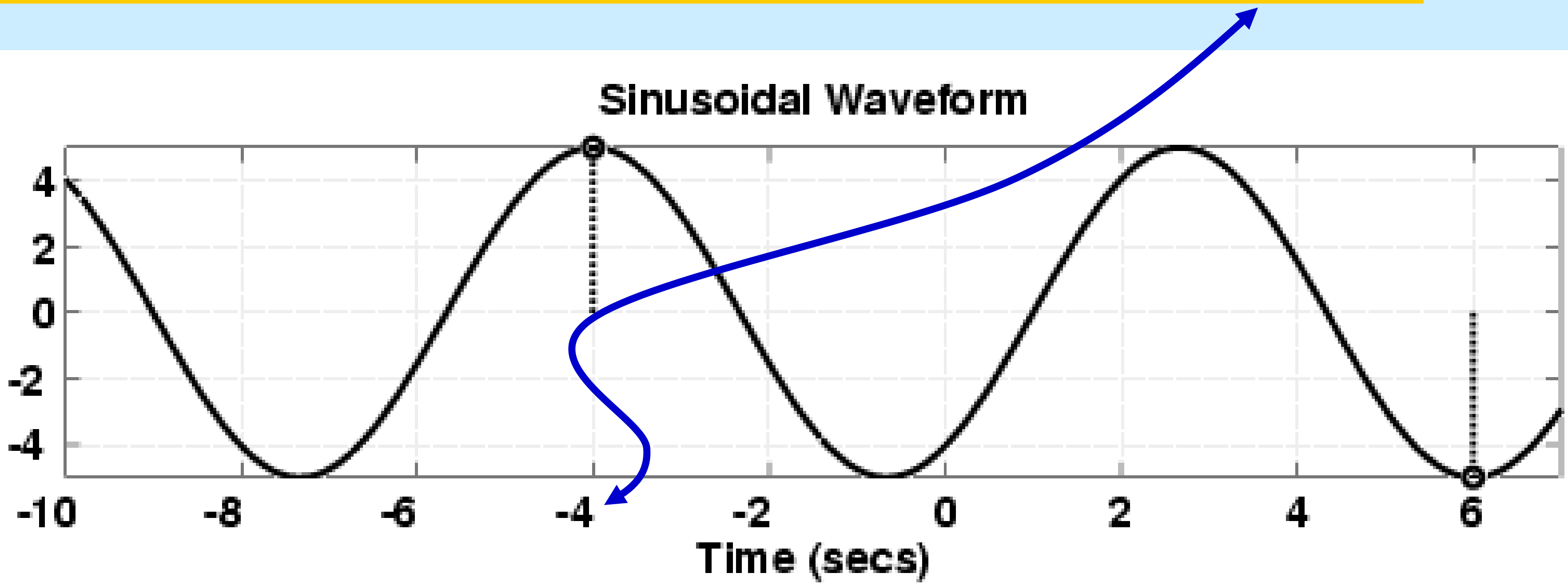
$$x(t-t_m) = A\cos(\omega(t-t_m))$$

- Then the t=0 point moves to $t=t_m$

- Peak value of $\cos(\omega(t\text{-}t_m))$ is now at $t=t_m$

TIME-SHIFTED SINUSOID

$$x(t+4) = 5\cos(0.3\pi(t+4)) = 5\cos(0.3\pi(t-(-4)))$$



PHASE <--> TIME-SHIFT

- Equate the formulas:

$$A\cos(\omega(t - t_m)) = A\cos(\omega t + \varphi)$$

- and we obtain:

$$-\omega t_m = \varphi$$

- or,

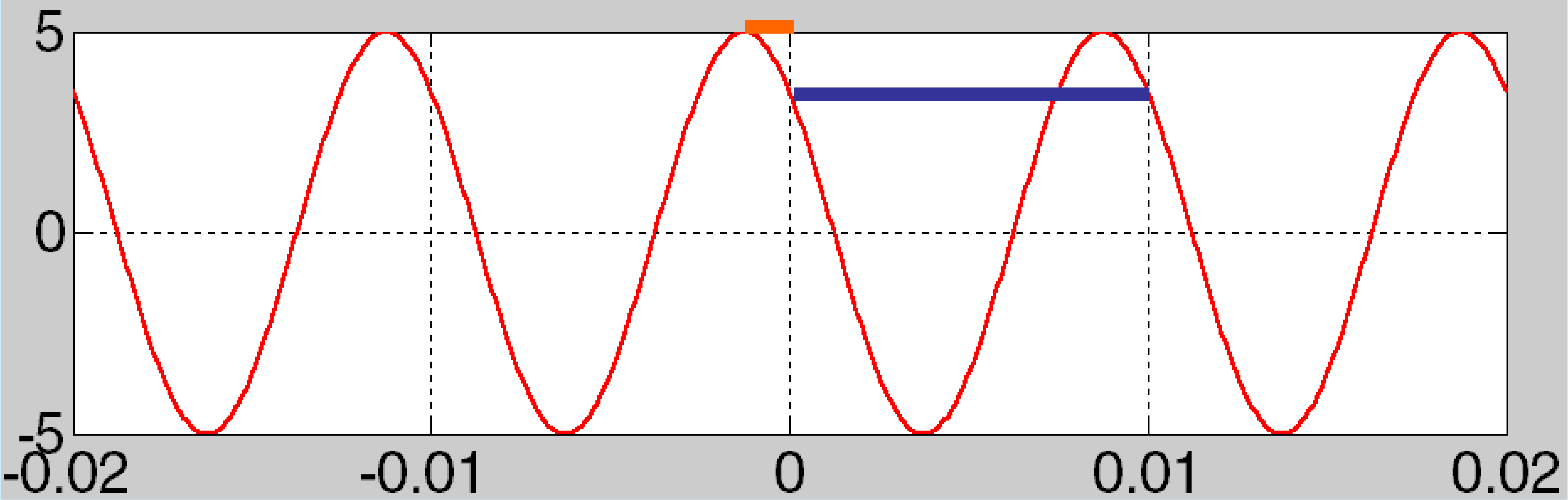
$$t_m = -\frac{\varphi}{\omega}$$

SINUSOID from a PLOT

- Measure the period, T
  - Between peaks or zero crossings
  - Compute frequency: $\omega = 2\pi/T$
- Measure time of a peak: t_m
  - Compute phase: $\phi = -\omega\, t_m$
- Measure height of positive peak: A

3 steps

(A, ω, ϕ) from a PLOT



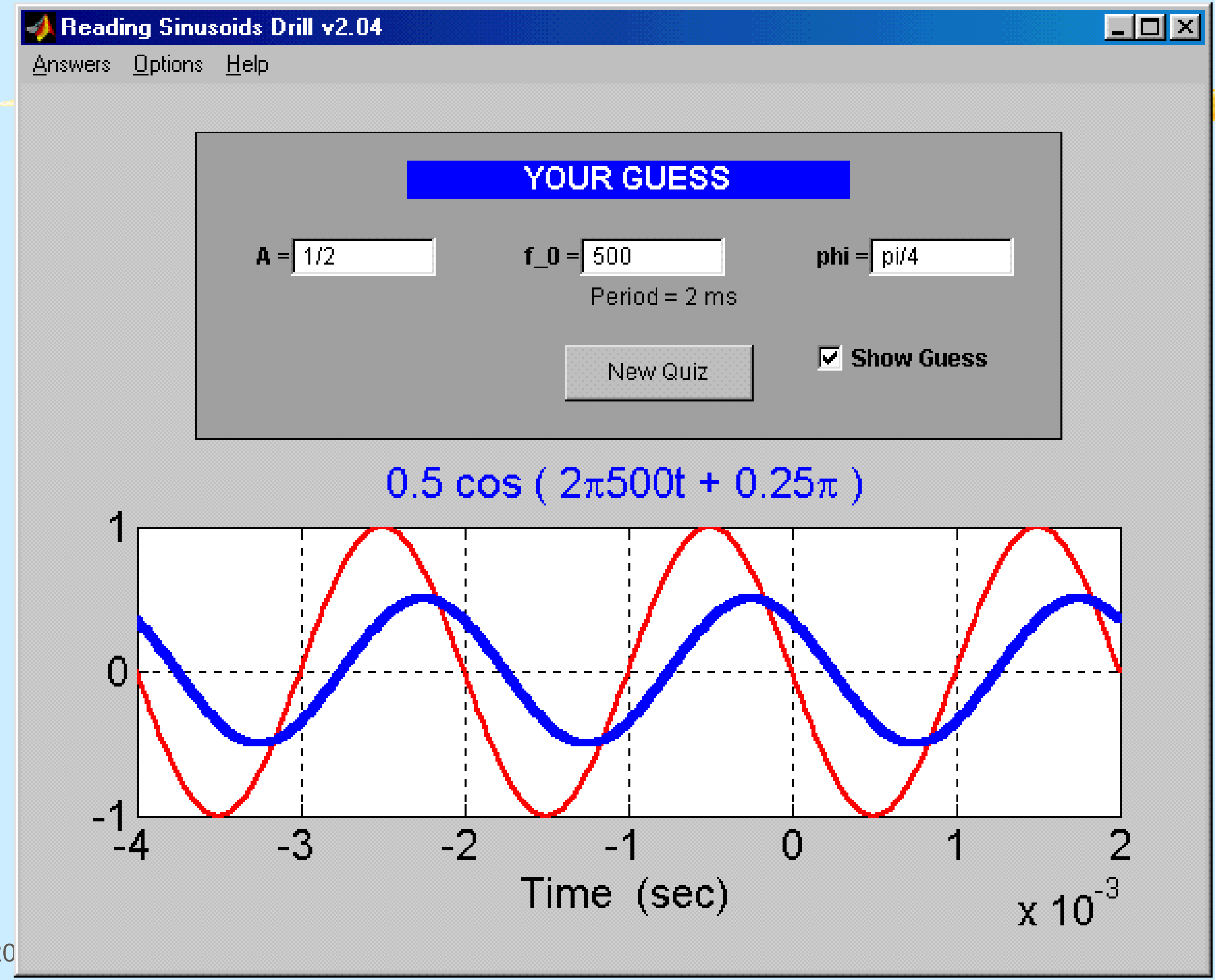
$$T = \frac{0.01\,\text{sec}}{1\,\text{period}} = \frac{1}{100}$$

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{0.01} = 200\pi$$

$$t_m = -0.00125\,\text{sec}$$

$$\varphi = -\omega t_m = -(200\pi)(t_m) = 0.25\pi$$

SINE DRILL (MATLAB GUI)



PHASE

- The cosine signal is periodic
  - Period is 2π

$$A\cos(\omega t+\varphi+2\pi)=A\cos(\omega t+\varphi)$$

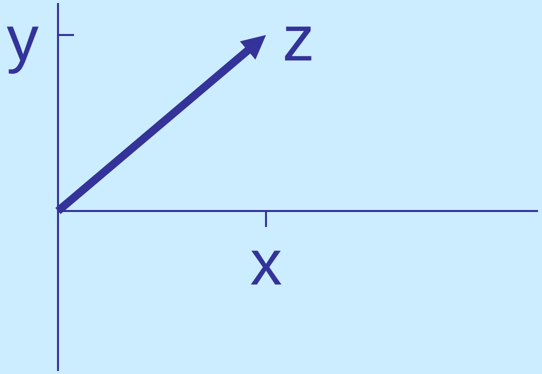
  - Thus adding any multiple of 2π leaves x(t) unchanged

$$\text{if} \quad t_m=\frac{-\varphi}{\omega}, \quad \text{then}$$

$$t_{m_2}=\frac{-(\varphi+2\pi)}{\omega}=\frac{-\varphi}{\omega}-\frac{2\pi}{\omega}=t_m-T$$

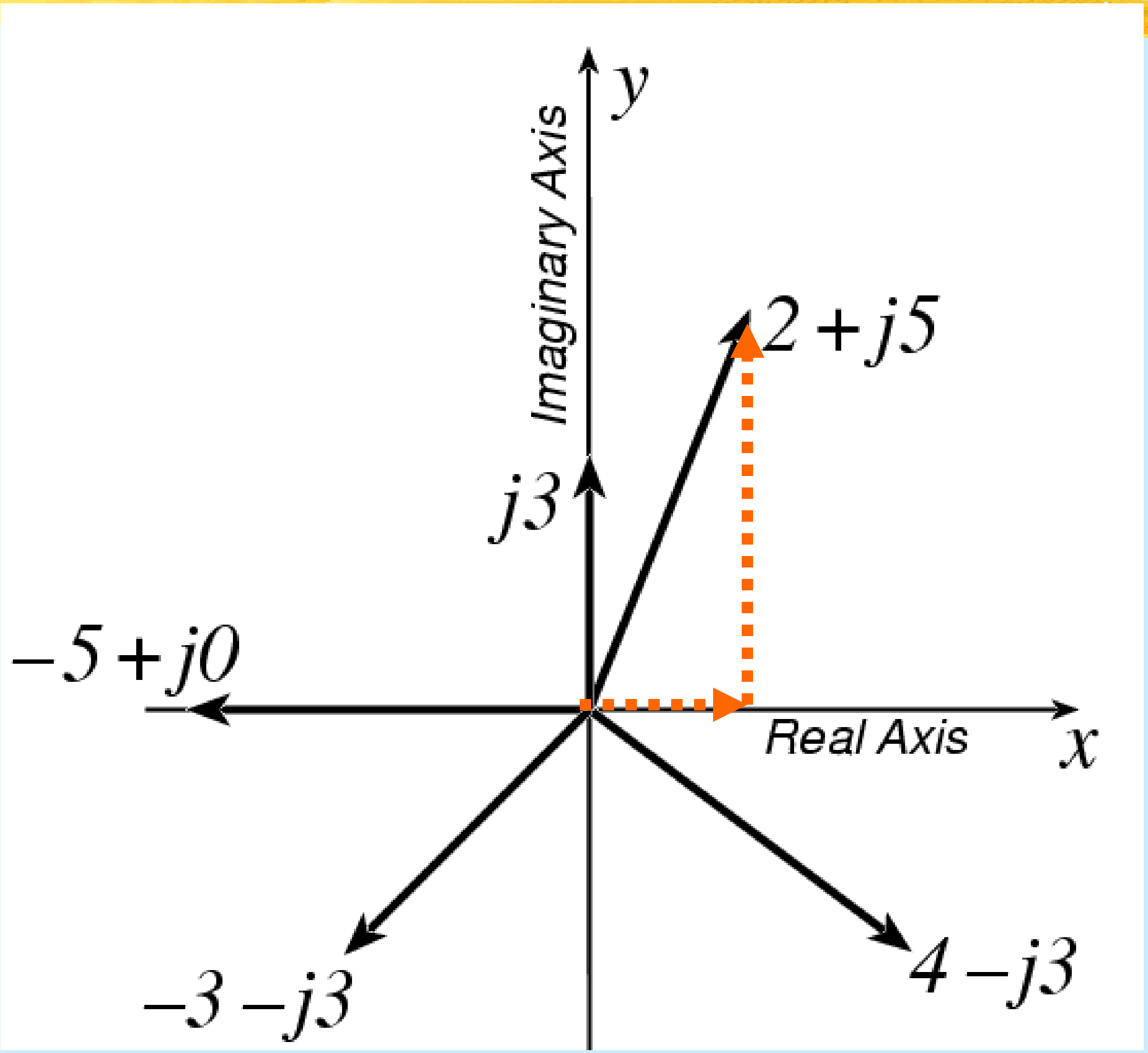
COMPLEX NUMBERS

- To solve: $z^2 = -1$
  - $z = \boldsymbol{j}$
  - Math and Physics use $z = \boldsymbol{i}$
- Complex number: $z = x + \boldsymbol{j}\, y$

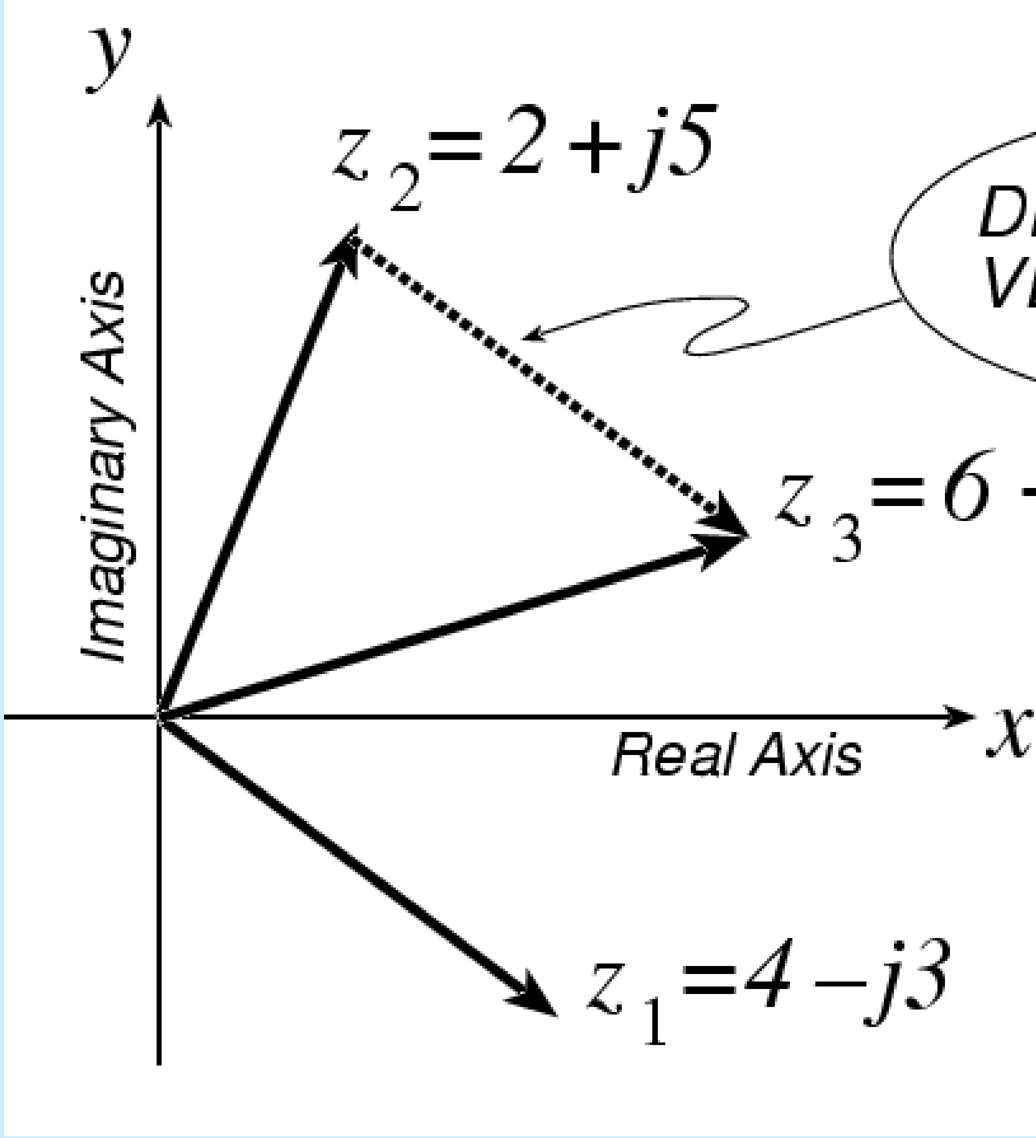


Cartesian coordinate system

PLOT COMPLEX NUMBERS



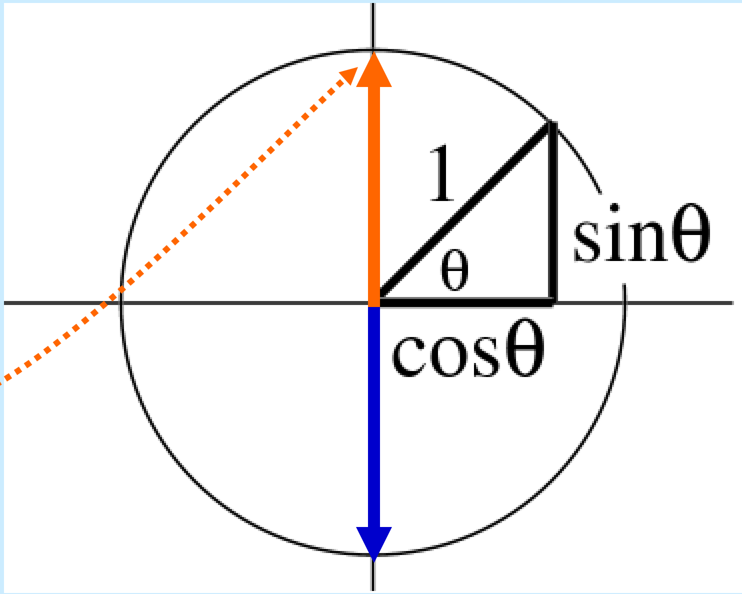
COMPLEX ADDITION = VECTOR Addition



$$
\begin{aligned}
z_3 &= z_1 + z_2 \\
&= (4 - j3) + (2 + j5) \\
&= (4 + 2) + j(-3 + 5) \\
&= 6 + j2
\end{aligned}
$$

*** POLAR FORM ***

- Vector Form
  - **Length** =1
  - **Angle** = θ
- Common Values
  - **j** has angle of 0.5π
  - –1 has angle of π
  - **– j** has angle of 1.5π
  - also, angle of **–j could** be –0.5π = 1.5π –2π
  - because the PHASE is **AMBIGUOUS**

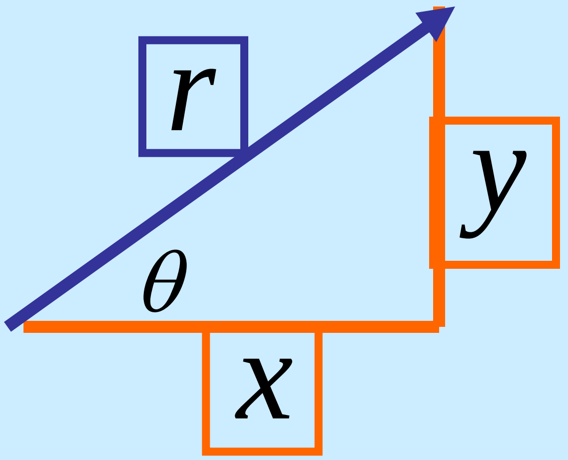


POLAR <--> RECTANGULAR

- Relate (x,y) to (r,θ)

$$r^2 = x^2 + y^2$$

$$\theta = \mathrm{Tan}^{-1}\left(\frac{y}{x}\right)$$



$$x = r\cos\theta$$

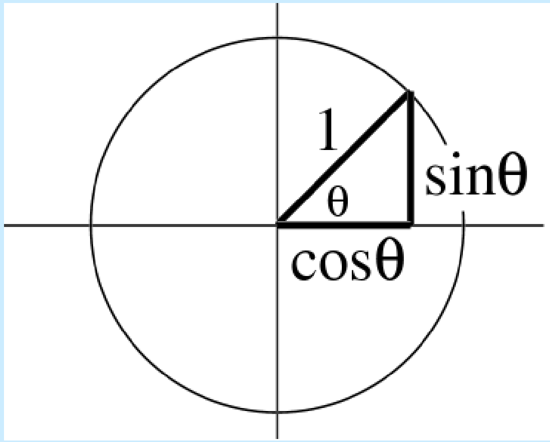
$$y = r\sin\theta$$

Most calculators do Polar-Rectangular

Need a notation for POLAR FORM

Euler's FORMULA

- **Complex Exponential**
  - Real part is cosine
  - Imaginary part is sine
  - Magnitude is one



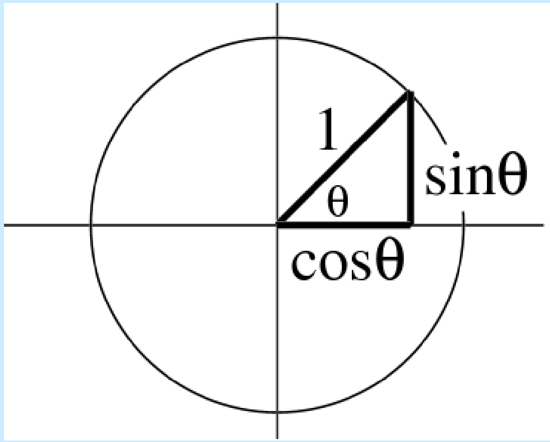
$$e^{j\theta} = \cos(\theta) + j\sin(\theta)$$

$$re^{j\theta} = r\cos(\theta) + jr\sin(\theta)$$

COMPLEX EXPONENTIAL

$$e^{j\omega t} = \cos(\omega t) + j\sin(\omega t)$$

- Interpret this as a **Rotating Vector**
  - $\theta = \omega t$
  - Angle changes vs. time
  - ex: $\omega = 20\pi$ rad/s
  - Rotates 0.2π in 0.01 secs



$$e^{j\theta} = \cos(\theta) + j\sin(\theta)$$

cos = REAL PART

Real Part of Euler's

$$\cos(\omega t) = \Re e\{e^{j\omega t}\}$$

General Sinusoid

$$x(t) = A\cos(\omega t + \varphi)$$

So,

$$A\cos(\omega t + \varphi) = \Re e\{Ae^{j(\omega t + \varphi)}\}$$
$$= \Re e\{Ae^{j\varphi}e^{j\omega t}\}$$

REAL PART EXAMPLE

$$A\cos(\omega t + \varphi) = \Re e\left\{Ae^{j\varphi}e^{j\omega t}\right\}$$

Evaluate:

$$x(t) = \Re e\left\{-3je^{j\omega t}\right\}$$

Answer:

$$x(t) = \Re e\left\{(-3j)e^{j\omega t}\right\}$$

$$= \Re e\left\{3e^{-j0.5\pi}e^{j\omega t}\right\} = 3\cos(\omega t - 0.5\pi)$$

COMPLEX AMPLITUDE

General Sinusoid

$$x(t) = A\cos(\omega t + \varphi) = \Re e\left\{Ae^{j\varphi}e^{j\omega t}\right\}$$

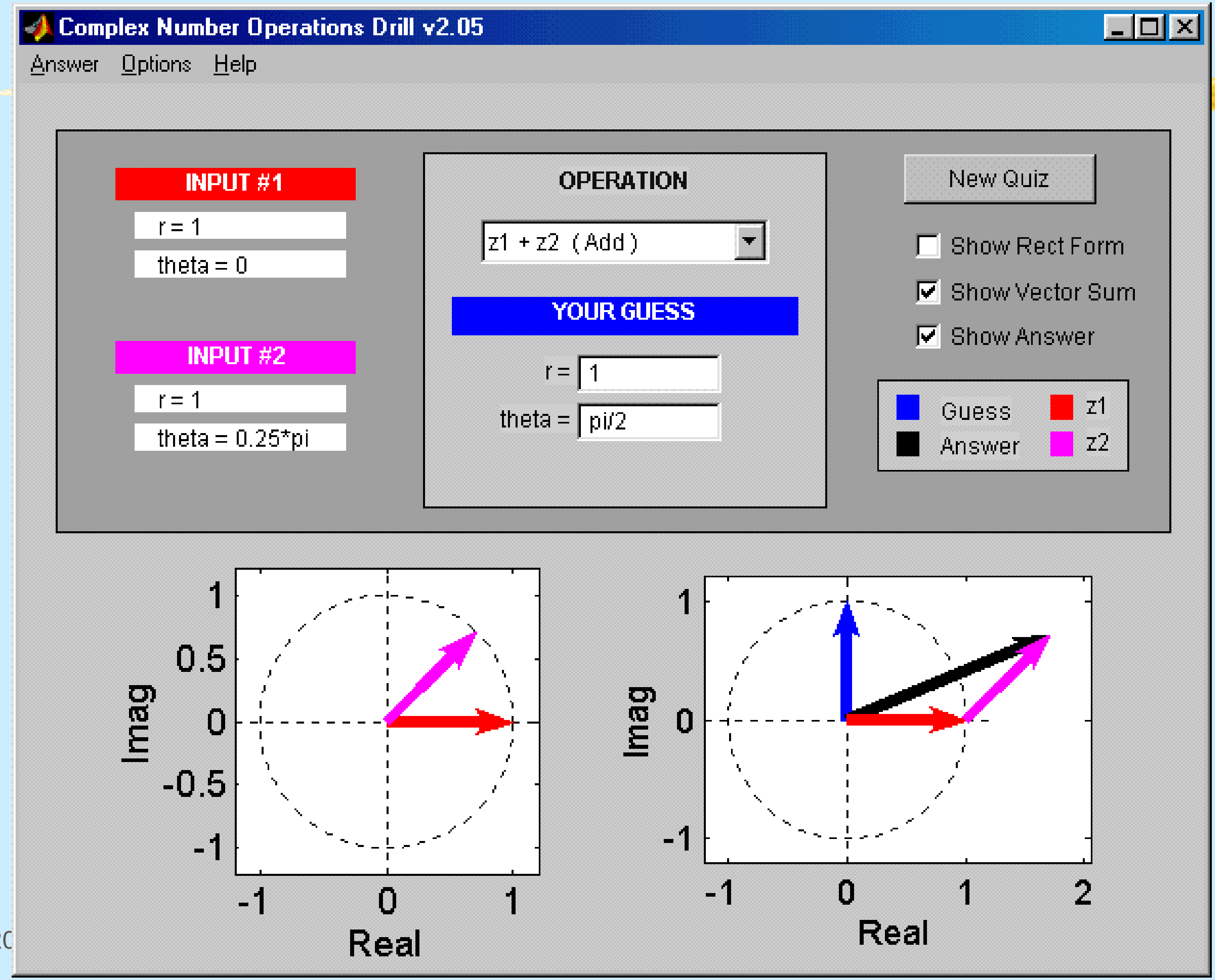
Complex AMPLITUDE = X

$$z(t) = Xe^{j\omega t} \qquad X = Ae^{j\varphi}$$

Then, any Sinusoid = REAL PART of $Xe^{j\omega t}$

$$x(t) = \Re e\left\{Xe^{j\omega t}\right\} = \Re e\left\{Ae^{j\varphi}e^{j\omega t}\right\}$$

Z DRILL (Complex Arith)



AVOID Trigonometry

- Algebra, even complex, is EASIER !!!
- Can you recall $\cos(\theta_1+\theta_2)$?
- Use: real part of $e^{j(\theta_1+\theta_2)} = \cos(\theta_1+\theta_2)$

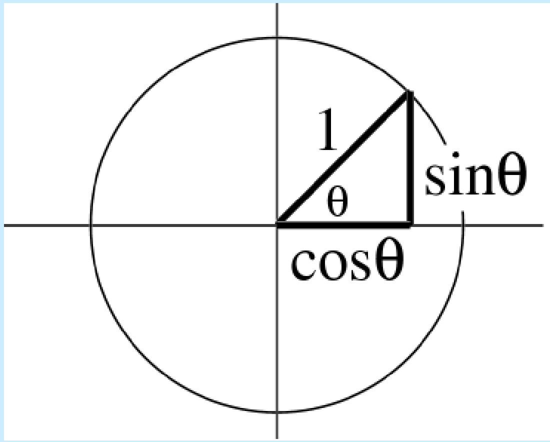
$$e^{j(\theta_1+\theta_2)} = e^{j\theta_1}e^{j\theta_2}$$

$$= (\cos\theta_1 + j\sin\theta_1)(\cos\theta_2 + j\sin\theta_2)$$

$$= \boxed{(\cos\theta_1\cos\theta_2 - \sin\theta_1\sin\theta_2)} + j(\ldots)$$

Euler's FORMULA

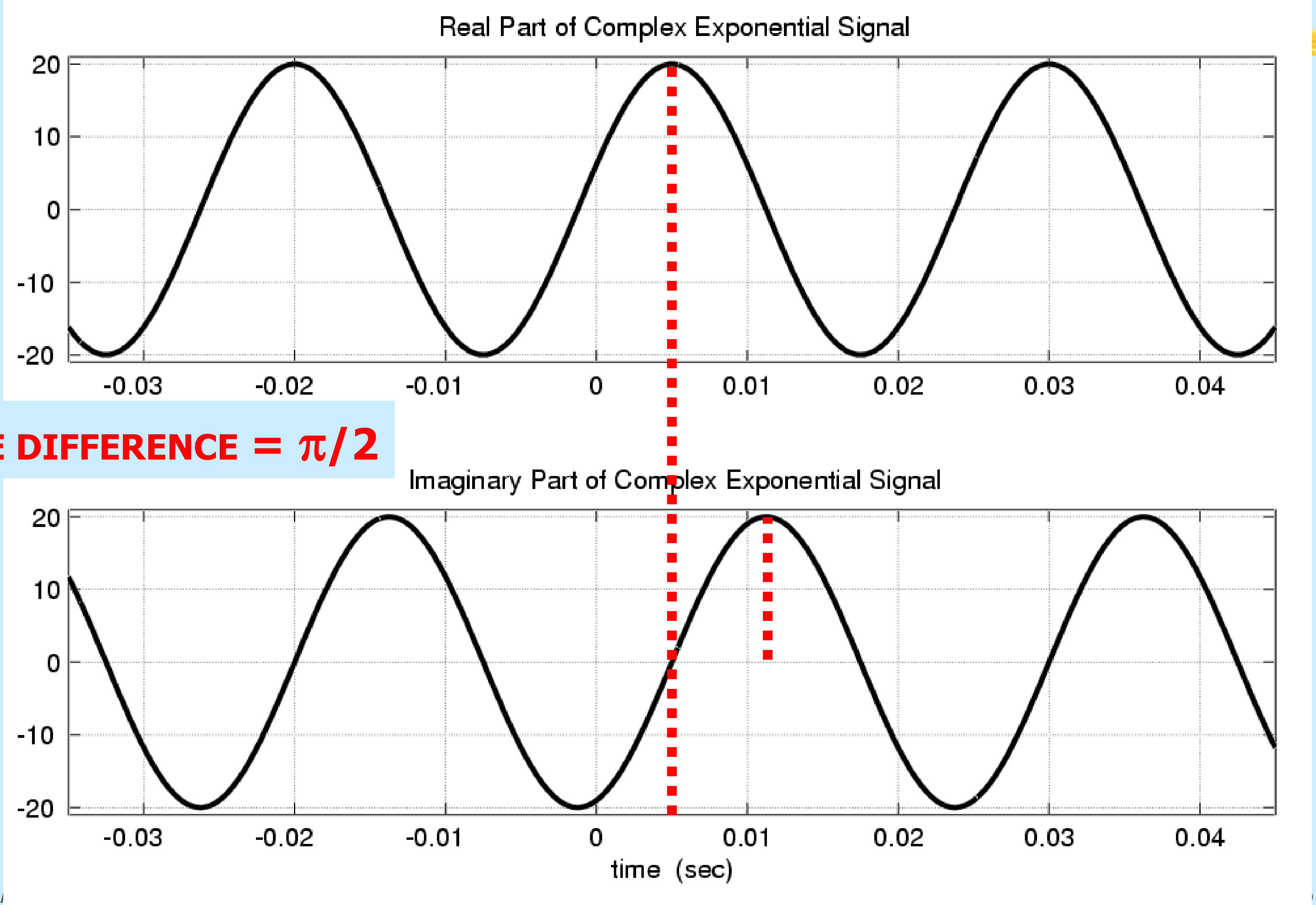
- Complex Exponential
  - Real part is cosine
  - Imaginary part is sine
  - Magnitude is one



$$e^{j\theta} = \cos(\theta) + j\sin(\theta)$$

$$e^{j\omega t} = \cos(\omega t) + j\sin(\omega t)$$

Real & Imaginary Part Plots

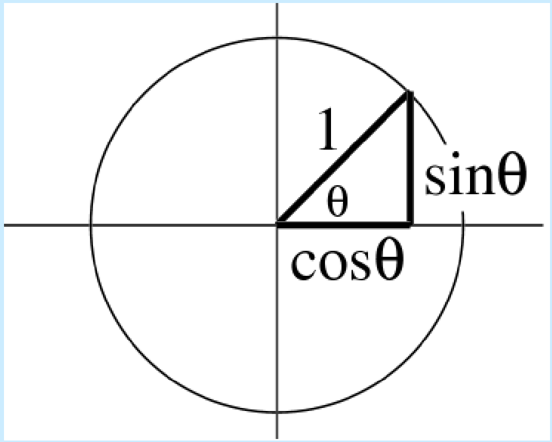


PHASE DIFFERENCE = $\pi/2$

COMPLEX EXPONENTIAL

$$e^{j\omega t} = \cos(\omega t) + j\sin(\omega t)$$

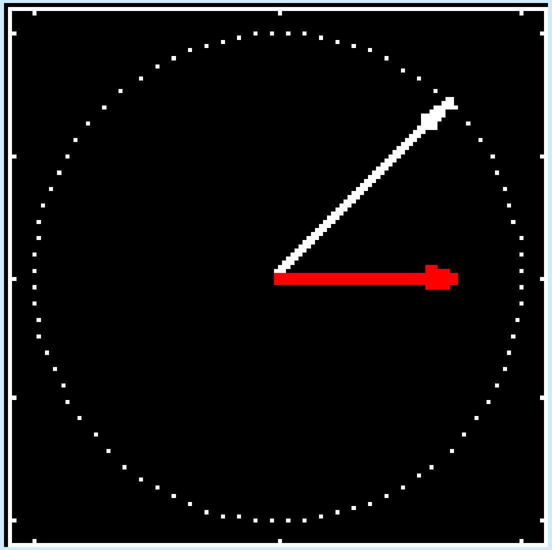
- Interpret this as a **Rotating Vector**
  - $\theta = \omega t$
  - Angle changes vs. time
  - ex: $\omega = 20\pi$ rad/s
  - Rotates 0.2π in 0.01 secs

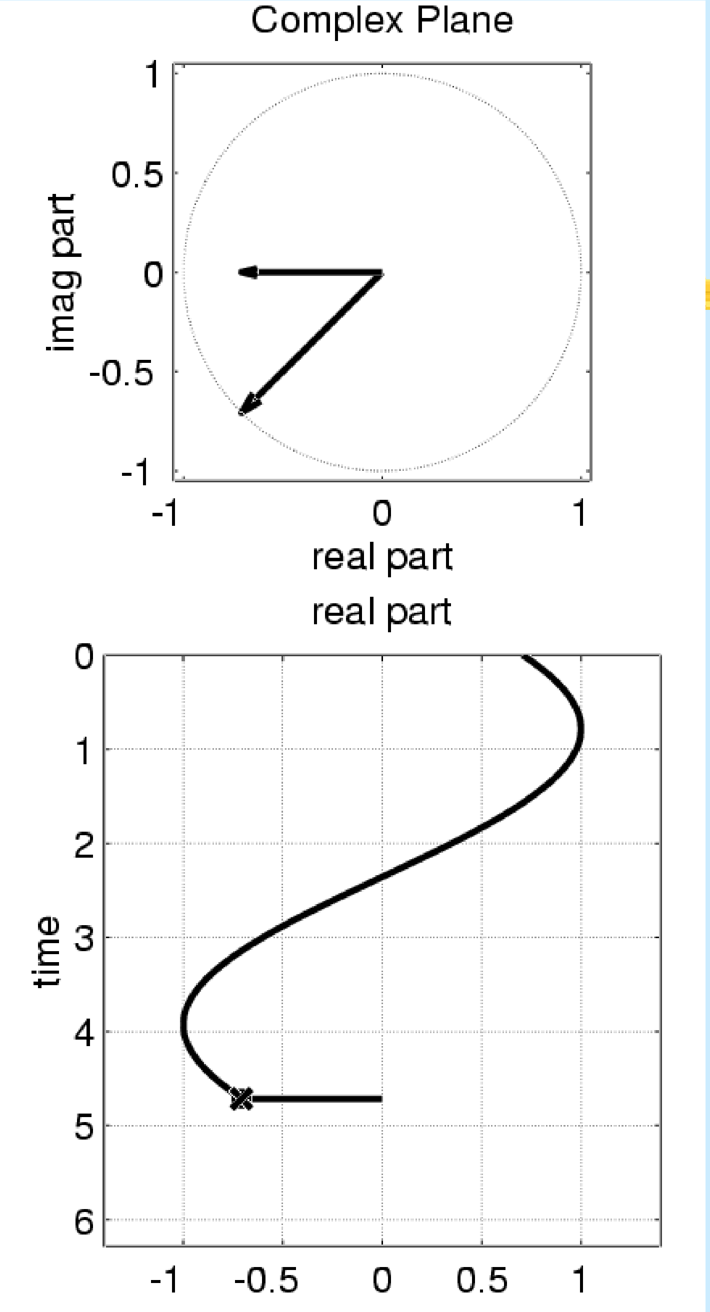


$$e^{jq} = \cos(q) + j\sin(q)$$

Rotating Phasor

See Demo on CD-ROM
Chapter 2





Cos = REAL PART

Real Part of Euler's

$$\cos(wt) = \hat{A}e\left\{e^{jwt}\right\}$$

General Sinusoid

$$x(t) = A\cos(wt + j)$$

So,

$$A\cos(wt + j) = \hat{A}e\left\{Ae^{j(wt+j)}\right\}$$

$$= \hat{A}e\left\{Ae^{jj}e^{jwt}\right\}$$

COMPLEX AMPLITUDE

General Sinusoid

$$x(t) = A\cos(\omega t + \varphi) = \Re e\left\{ A e^{j\varphi} e^{j\omega t} \right\}$$

Sinusoid = REAL PART of $(Ae^{j\phi})e^{j\omega t}$

$$x(t) = \Re e\left\{ X e^{j\omega t} \right\} = \Re e\left\{ z(t) \right\}$$

Complex AMPLITUDE = X

$$z(t) = X e^{j\omega t} \qquad X = A e^{j\varphi}$$

POP QUIZ: Complex Amp

- Find the COMPLEX AMPLITUDE for:

$$x(t) = \sqrt{3}\cos(77\pi t + 0.5\pi)$$

- Use EULER's FORMULA:

$$x(t) = \Re e\left\{\sqrt{3}e^{j(77\pi t + 0.5\pi)}\right\}$$

$$= \Re e\left\{\sqrt{3}e^{j0.5\pi}e^{j77\pi t}\right\}$$

$$X = \sqrt{3}e^{j0.5\pi}$$

WANT to ADD SINUSOIDS

- ALL SINUSOIDS have SAME FREQUENCY
- HOW to GET {Amp,Phase} of RESULT ?

$$x_1(t) = 1.7\cos(2\pi(10)t + 70\pi/180)$$

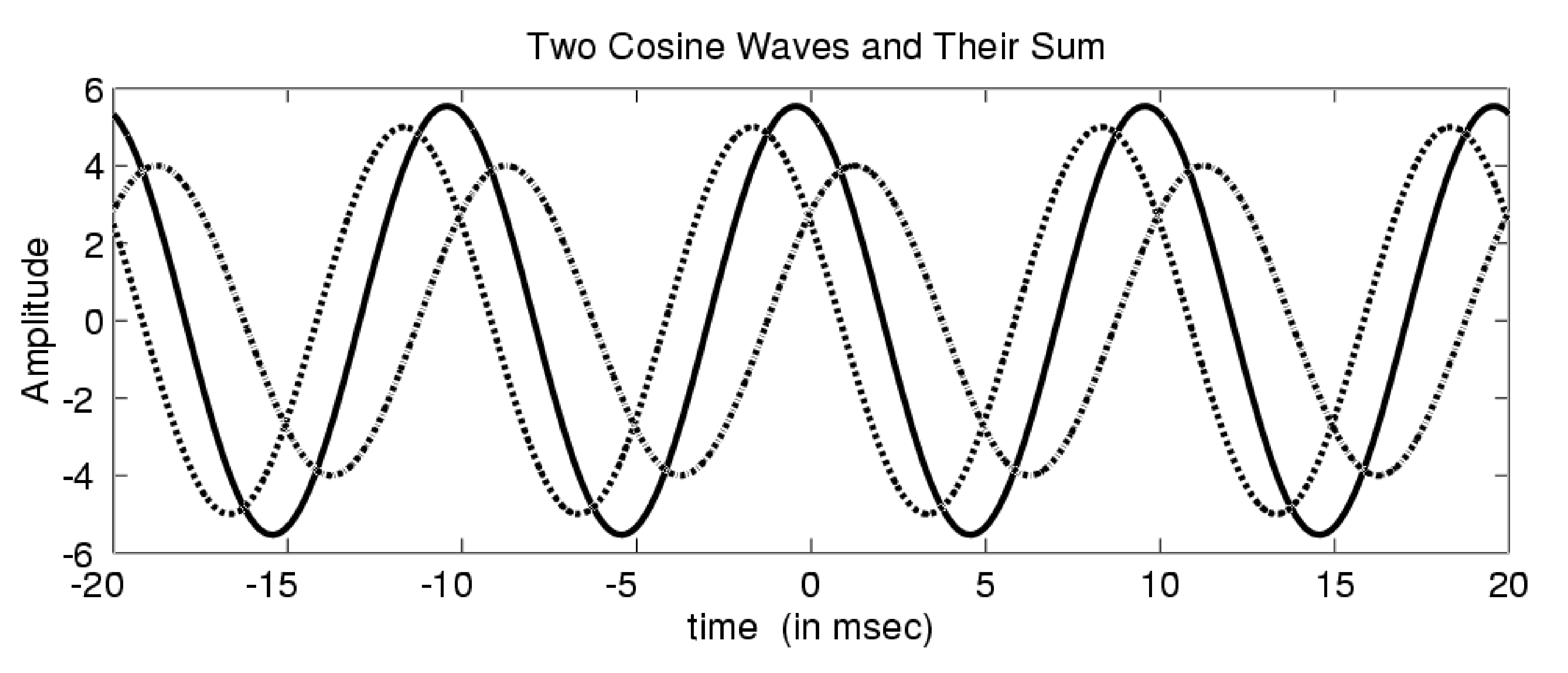
$$x_2(t) = 1.9\cos(2\pi(10)t + 200\pi/180)$$

$$x_3(t) = x_1(t) + x_2(t)$$

$$= 1.532\cos(2\pi(10)t + 141.79\pi/180)$$

ADD SINUSOIDS

- Sum Sinusoid has **SAME** Frequency



PHASOR ADDITION RULE

$$x(t) = \sum_{k=1}^{N} A_k \cos(\omega_0 t + \phi_k)$$

$$= A \cos(\omega_0 t + \phi)$$

Get the new complex amplitude by complex addition

$$\sum_{k=1}^{N} A_k e^{j\phi_k} = A e^{j\phi}$$

Phasor Addition Proof

$$\sum_{k=1}^{N} A_k \cos(\omega_0 t + \phi_k) = \sum_{k=1}^{N} \Re e\left\{A_k e^{j(\omega_0 t + \phi_k)}\right\}$$

$$= \Re e\left\{\sum_{k=1}^{N} A_k e^{j\phi_k} e^{j\omega_0 t}\right\}$$

$$= \Re e\left\{\left(\sum_{k=1}^{N} A_k e^{j\phi_k}\right) e^{j\omega_0 t}\right\}$$

$$= \Re e\left\{\left(A e^{j\phi}\right) e^{j\omega_0 t}\right\} = A\cos(\omega_0 t + \phi)$$

POP QUIZ: Add Sinusoids

- ADD THESE 2 SINUSOIDS:

$$x_1(t) = \cos(77\pi t)$$

$$x_2(t) = \sqrt{3}\cos(77\pi t + 0.5\pi)$$

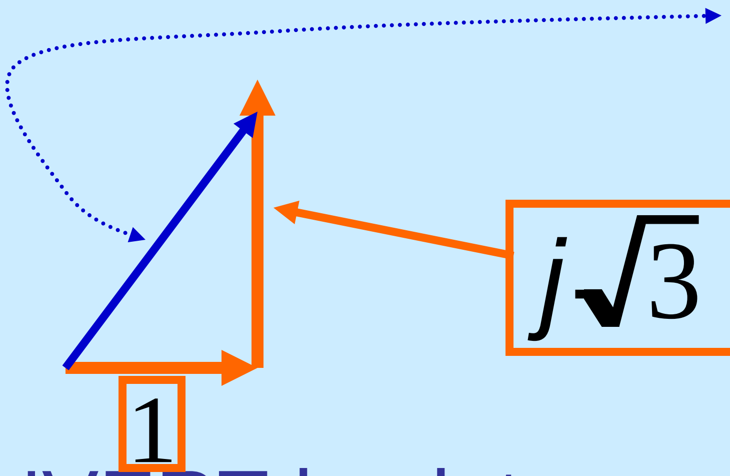
- COMPLEX ADDITION:

$$1e^{j0} + \sqrt{3}e^{j0.5\pi}$$

POP QUIZ (answer)

- COMPLEX ADDITION:

$$1 + j\sqrt{3} = 2e^{j\pi/3}$$

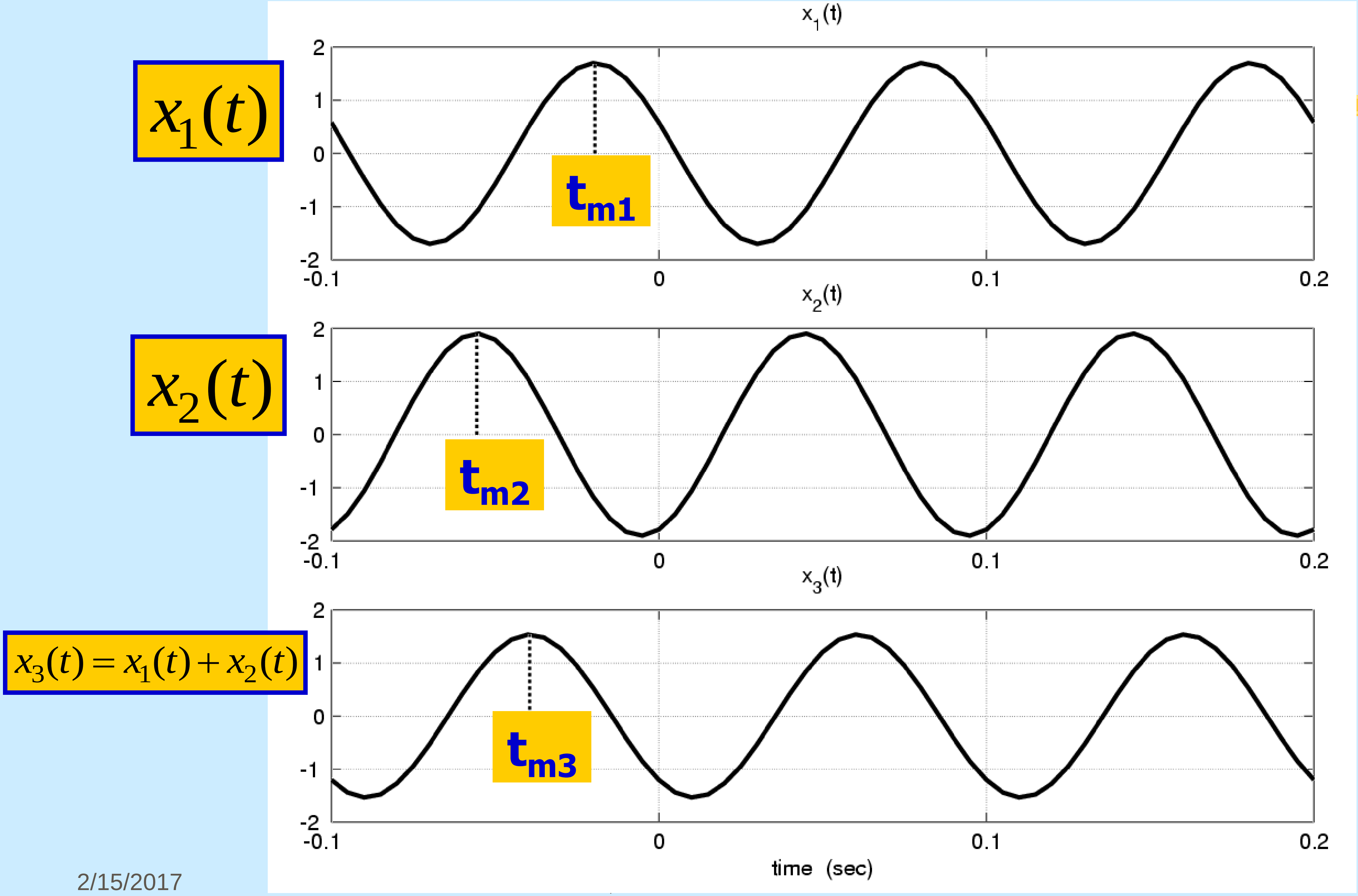


$$j\sqrt{3} = \sqrt{3}e^{j\,0.5p}$$

- CONVERT back to cosine form:

$$x_3(t) = 2\cos(77\pi t + \frac{\pi}{3})$$

ADD SINUSOIDS EXAMPLE

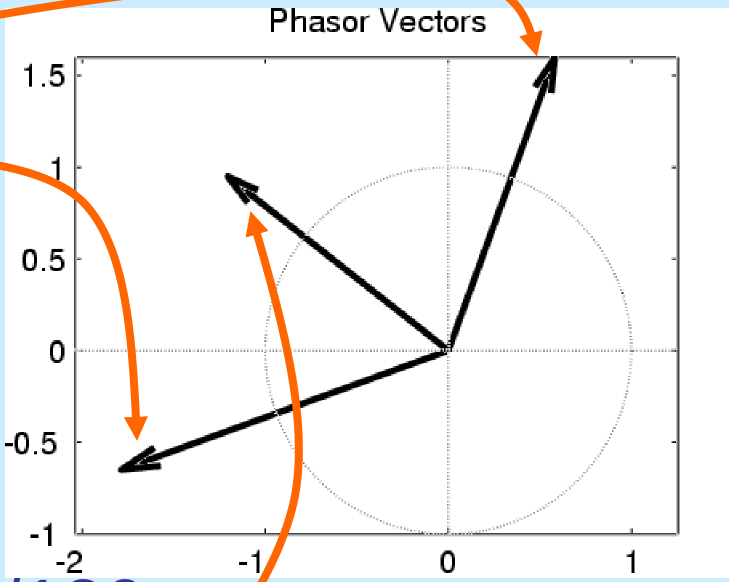


Convert Time-Shift to Phase

- Measure peak times:
  - t_{m1}=-0.0194, t_{m2}=-0.0556, t_{m3}=-0.0394
- Convert to phase (T=0.1)
  - $\phi_1 = -\omega t_{m1} = -2\pi(t_{m1}/T) = 70\pi/180$,
  - $\phi_2 = 200\pi/180$
- Amplitudes
  - A_1=1.7, A_2=1.9, A_3=1.532

Phasor Add: Numerical

- Convert Polar to Cartesian
  - $X_1 = 0.5814 + j1.597$
  - $X_2 = -1.785 - j0.6498$
  - sum =
  - $X_3 = -1.204 + j0.9476$
- Convert back to Polar
  - $X_3 = 1.532$ at angle $141.79\pi/180$
  - This is the sum



ADD SINUSOIDS

$$x_1(t) = 1.7\cos(2\pi(10)t + 70\pi/180)$$

$$x_2(t) = 1.9\cos(2\pi(10)t + 200\pi/180)$$

$$x_3(t) = x_1(t) + x_2(t)$$
$$= 1.532\cos(2\pi(10)t + 141.79\pi/180)$$

VECTOR (PHASOR) ADD