4.1 $x(t) = 10\cos(880\pi t + \phi)$

$x[n] = x(nT_s) = 10\cos(880\pi n T_s + \phi)$

$T_s = 0.0001$

a. How many samples over one period.

$$880\pi n T_s \leq 2\pi$$

$$n \leq \frac{2}{0.088} \leq 22.73$$

$n = 23$ samples $\leftarrow$ $0, 1, \ldots 22$

(b) $y(t) = 10\cos(\omega_0 t + \phi)$

Find a freq. $\omega_0 > 880\pi$ such that $y(nT_s) = x(nT_s)$ for all n.

$$\omega_0 n T_s = 880 n\pi T_s + 2\pi l n$$

$$\omega_0 = 880\pi + \frac{2\pi l}{T_s}$$

$$\omega_0 = 880\pi + \frac{2\pi}{T_s}$$

$$= 20880\pi$$

(c) $$20880\pi n T_s \leq 2\pi$$

$$n < \frac{2}{2.088} < 1$$ 1 sample (0^{th})

4.2 $x(t) = 7\sin(11\pi t)$

a. $f_s = 10$ sample/sec.

$x[n] = ?$

$$x[n] = 7\sin\left(11\pi.\frac{n}{10}\right)$$

$$x[n] = 7\cos\left(\frac{11\pi n}{10} - \frac{\pi}{2}\right)$$

b $f_s = 5$

$$x(n] = 7\cos\left(\frac{11\pi n}{5} - \pi/2\right)$$

c $f_s = 15$

$$x[n] = 7\cos\left(\frac{11\pi n}{15} - \pi/2\right)$$

4.3 $x[n] = 2.2 \cdot \cos(0.3\pi n - \pi/3)$

↳ obtained from $x(t) = A\cos(2\pi f_0 t + \phi)$

$f_s = 6000\,Hz$

Determine three diff. continuous time signals.

$$\frac{2\pi f_0 n}{f_s} = 0.3\pi n$$

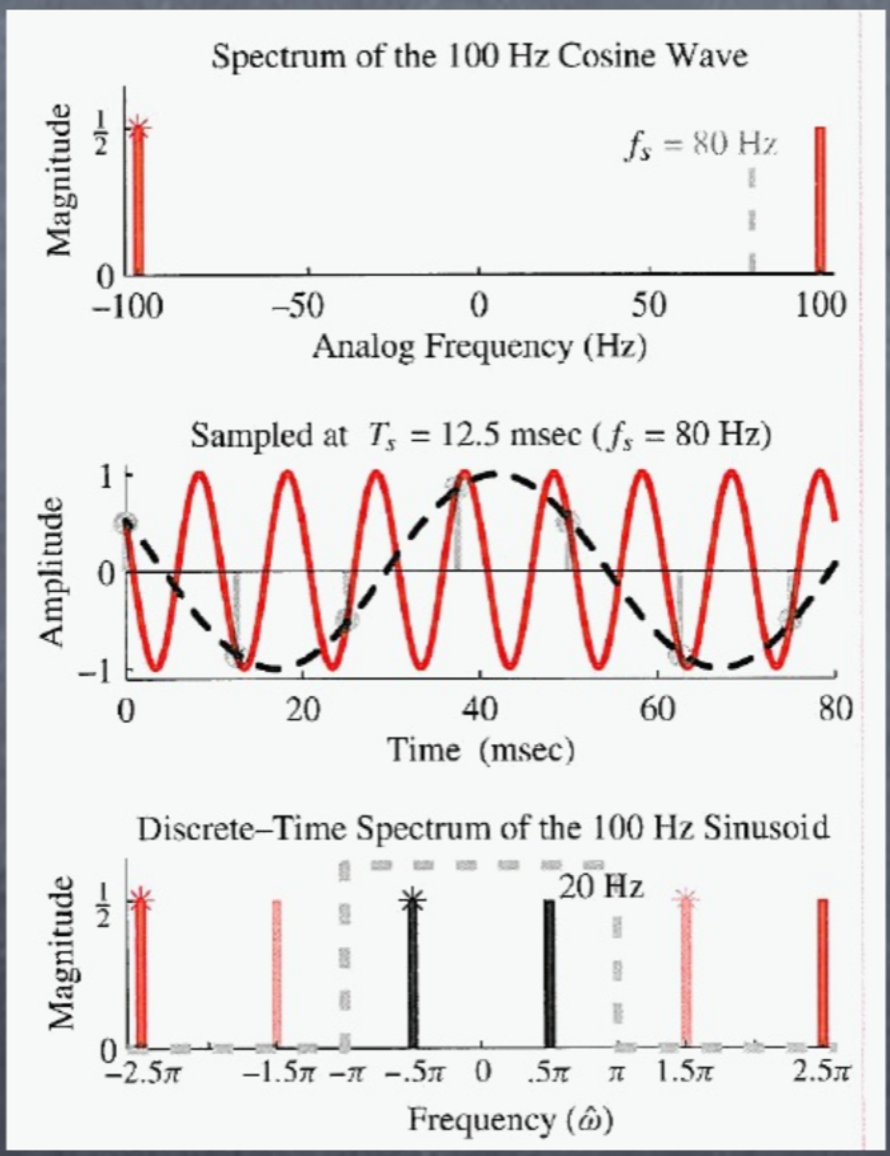
$$\frac{2\pi \bar{f}_0 n}{f_s} = 0.3\pi n + 2\pi n$$

$$\frac{2\pi \bar{\bar{f}}_0 n}{f_s} = 0.3\pi n - 2\pi n$$

$f_0 = 900\,Hz$ $\quad x(t) = 2.2\cos(1800\pi t - \pi/3)$

$\bar{f}_0 = 6900\,Hz$ $\quad x(t) = 2.2\cos(2\pi(6900)t - \frac{\pi}{3})$

$\bar{\bar{f}}_0 = -5100\,Hz$ $\quad x(t) = 2.2\cos(2\pi(-5100)t - \frac{\pi}{3})$



Normalized radian freq: $\omega = \pm 2\pi \left(\frac{f_0}{f_s}\right)$

$\omega = 2\pi \cdot \frac{100}{80} = \pm 2.5\pi \quad \begin{pmatrix} 2.5\pi + 2\pi \ell \\ -2.5\pi + 2\pi \ell \end{pmatrix}$

$\ell = 0, \ldots$

$0.5\pi = 2\pi \cdot \frac{f_0}{f_s \to 80} = \quad f_0 = \underline{20\text{Hz}}$

4.4 $x(t) = \left[10 + \cos\left(2\pi(2000t)\right)\right]\cos\left(2\pi(10^4)t\right)$

(a) $$10 + \left[\frac{1}{2}\cdot e^{j2\pi(2000)t} + \frac{1}{2}\cdot e^{-j2\pi(2000)t}\right]$$

$$\left[\frac{1}{2}\cdot e^{-j10^4 2\pi t} + \frac{1}{2}\cdot e^{+j10^4 2\pi t}\right]$$

$$= 5\cdot e^{j2\pi\cdot 10^4 t} + 5\cdot e^{-j2\pi\cdot 10^4 t} + \frac{1}{4}\cdot e^{-j2000(2\pi)t}$$

$$+ \frac{1}{4}\cdot e^{-j2\pi(12000)t} + \frac{1}{4}\cdot e^{j2\pi(8000)t}$$

$$+ \frac{1}{4}\cdot e^{-j2\pi(8000)t}$$

8k, -8k, -12k, 10k, -10k

$f_f = 2000\,\text{Hz}$

$f_N > 12000 \times 2 \;\; > 24k$

4.5 (a) $x[n] = 10\cos(0.2\pi n - \pi/7)$

$f_s = 1000$ $\qquad |f_0| < 1000Hz$

$$2\pi \cdot f_0 \cdot \frac{n}{f_s} = 0.2\pi n$$

$$2\pi \bar{f_0} \cdot \frac{n}{f_s} = 0.2\pi n + 2\pi n$$

$$2\pi \bar{\bar{f_0}} \frac{n}{f_s} = 0.2\pi n - 2\pi n$$

$f_0 = 100 Hz$ $\qquad \cancel{f_0 = 1100 Hz}$ $\qquad f_0 = -900 Hz$

$$\underbrace{10\cos(2\pi \cdot 100t - \pi/7)}_{x_1(t)} \qquad \underbrace{10\cos\left(2\pi(-900) - \frac{\pi}{7}\right)}_{x_2(t)}$$

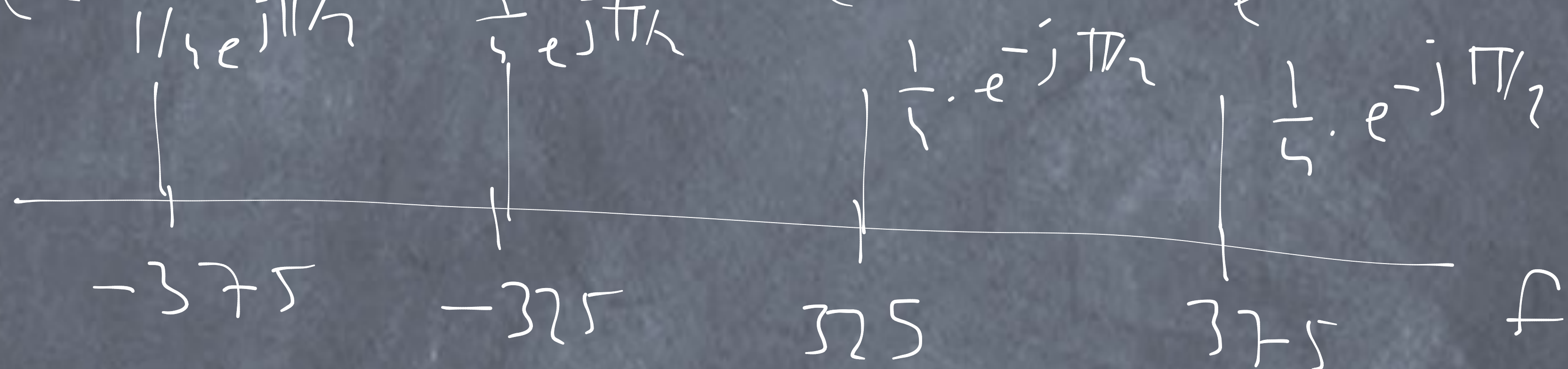
4.8 $x(t) = \cos(50\pi t)\cdot\sin(700\pi t)$

a. Draw two sided spectrum.

b. Determine the minimum sampling rate that can be used to sample $x(t)$ without aliasing for any of the component

$$\left(\frac{1}{2}\cdot e^{j50\pi t} + \frac{1}{2}\cdot e^{-j50\pi t}\right)\left(\frac{1}{2j}e^{j700\pi t} - \frac{1}{2j}e^{-j700\pi t}\right)$$

$$= \frac{1}{4j}\left(e^{j750\pi t} + e^{j650\pi t} - e^{-j650\pi t} - e^{-j750\pi t}\right)$$



$f_s > 375 \times 2 > 750\,Hz$