1. $\mathcal{F}\{2 \cdot e^{-t} u(t) + 3 \cdot e^{-2t} u(t)\}$

$$\frac{2}{1 + j\omega} + \frac{3}{2 + j\omega}$$

2. $f(t) = \begin{cases} 4 & -3 \le t \le 3 \\ 0 & \text{o.w.} \end{cases}$

$$f(t) = 4 P_3(t)$$

$$F(\omega) = \frac{8}{3\omega} \sin 3\omega$$

3. $g(t) = \begin{cases} 1 & 3 \le t \le 5 \\ 0 & \text{o.w.} \end{cases}$

$$p_1(t) = \begin{cases} 2 & -1 \le t \le 1 \\ 0 & \text{o.w.} \end{cases} \quad (\text{shift } + 4 \text{ units})$$

$$e^{-j4\omega} \frac{\sin \omega}{\omega} \cdot 2$$

$$G(\omega) = \int_3^5 1 \cdot e^{-j\omega t} dt$$

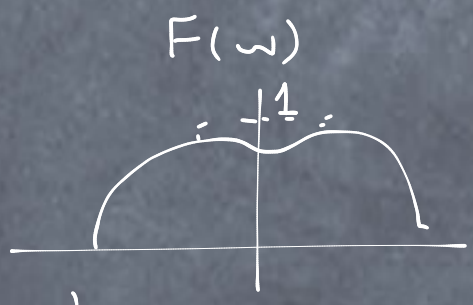
$$= \int_3^5 \frac{e^{-j\omega t}}{-j\omega} = \frac{e^{-j5\omega} - e^{-j3\omega}}{-j\omega}$$

$$e^{-j4\omega} \left(\frac{e^{-j\omega} - e^{j\omega}}{-j\omega} \right) = e^{-j4\omega} \cdot \frac{\sin \omega}{\omega} \cdot 2$$

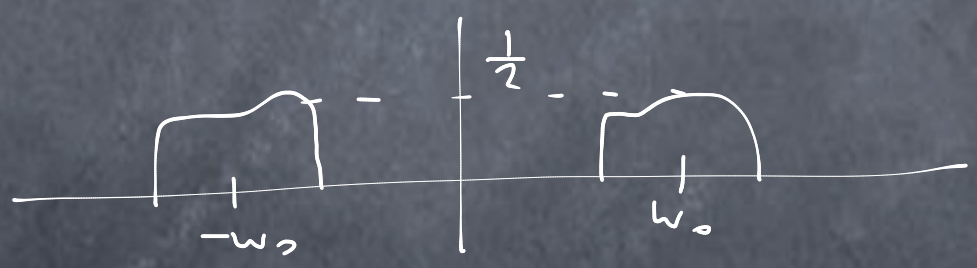
4. $f(t) \xleftrightarrow{F} F(\omega)$

$g(t) = f(t)\cos\omega_0 t$

$G(\omega)$



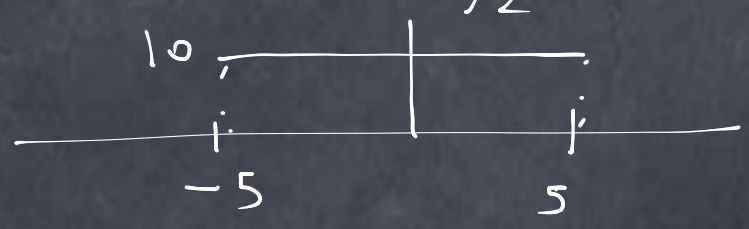
$$\frac{1}{2}\cdot e^{j\omega_0 t} f(t) + \frac{1}{2}\cdot e^{-j\omega_0 t} f(t)$$



5. Find inverse Fourier transform of

$$F(\omega) = 20\,\frac{\sin 5\omega}{\omega}$$

$$= 10\cdot\frac{\sin 5\omega}{\omega/2}$$



11.2 Shift Property

(a) $x(t) = \delta(t+1) + 2\delta(t) + \delta(t-1)$

$e^{jw} \cdot 1 + 2 + e^{-jw}$

$= e^{jw} \cdot 1 + 2 + e^{-jw}$

$= 2 + 2\cos w$

(b) $\dfrac{\sin(100\pi(t-2))}{\pi(t-2)}$

$\{u(w+100\pi) - u(w-100\pi)\}\, e^{-j2w_0}$

11.4 $x(t) = \begin{cases} 1 & 0 \le t < 4 \\ 0 & \text{o.t.w} \end{cases}$

$u(t+2) - u(t-2) \longrightarrow$ time shift

$\frac{\sin 2\omega}{\omega/2} \cdot e^{-j2\omega}$

b. $S(j\omega) = 4\pi\delta(\omega) + 2\pi\delta(\omega - 10\pi) +$

$2\pi\delta(\omega+10\pi)$ $2 + e^{j10\pi t} + e^{-j10\pi t}$

$= 2 + 2\cos(10\pi t)$

c. $R(j\omega) = \frac{j\omega}{4+2j\omega} = \frac{1}{2} - \frac{2}{4+2j\omega}$

$\frac{1}{2}\delta(t) - e^{-2t}u(t)$

d. $y(t) = \delta(t+1) + 2\delta(t) + \delta(t-1)$

$= e^{j\omega} + 2 + e^{-j\omega}$

$= 2 + 2\cos\omega$

11.3 $X(j\omega) = \frac{j\omega}{0.1+j\omega} e^{-j\omega 0.2} = \underbrace{X_1(j\omega) e^{-j0.2\omega}}_{\text{time shift}}$

$X_1(j\omega) = j\omega X_2(j\omega)$ derivative

$X_2(j\omega) = \frac{1}{0.1+j\omega} \Rightarrow x_2(t) = e^{-0.1t} u(t)$

$x_1(t) = \frac{d}{dt} x_2(t) = e^{-0.1t}\delta(t) - 0.1 e^{-0.1t} u(t)$

$x(t) = \delta(t-0.2) - 0.1 e^{-0.1(t-0.2)} u(t-0.2)$

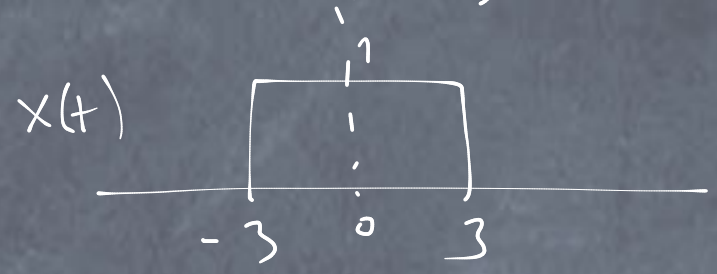
b. $X(j\omega) = 2 + 2\cos\omega = 2 + e^{-j\omega} + e^{j\omega}$

$= 2\delta(t) + \delta(t-1) + \delta(t+1)$

c. $X(j\omega) = \frac{1}{1+j\omega} - \frac{1}{2+j\omega}$

$x(t) = e^{-t} u(t) - e^{-2t} u(t)$

11.6 $x(t) = u(t+3)\,u(3-t)$ $X(w)$



$$X(jw) = \frac{\sin 3w}{w/2}$$

⑤ $\sin(4\pi t)\sin(50\pi t) \xrightarrow{F}$