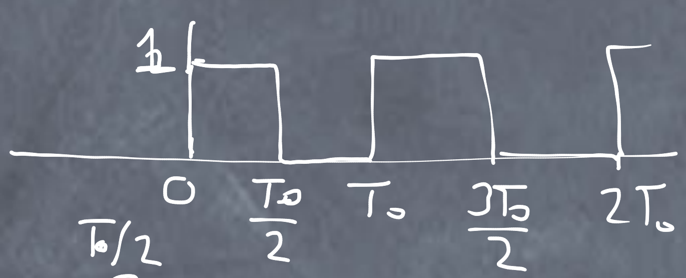
Ex:



$$a_0 = \frac{1}{T_0}\int_0^{T_0/2} 1 \, dt = \frac{1}{2}$$

$$a_k = \frac{1}{T_0}\int_0^{T_0/2} 1 \cdot e^{-j(2\pi/T_0)kt} \, dt$$

$$a_k = \frac{1}{T_0} \left. \frac{e^{-j(2\pi/T_0)kt}}{-j(2\pi/T_0)k} \right|_0^{T_0/2}$$

$$= \frac{1}{T_0} \frac{e^{-j(2\pi/T_0)k \cdot \frac{T_0}{2}} - e^0}{j(2\pi/T_0)k}$$

$$= \frac{e^{-j\pi k} - 1}{-j2\pi k} = \frac{(-1)^k - 1}{-j2\pi k}$$

k even $\rightarrow 0$

k odd $\rightarrow \frac{1}{j\pi k}$

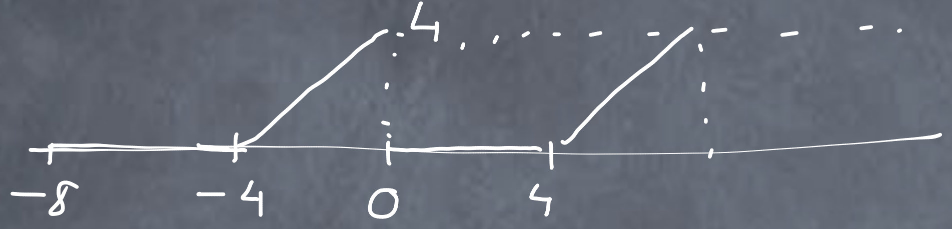
$k = 0 \rightarrow 1/2$

3.11

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{j(2\pi/8)kt}$$

$$a_k = \frac{1}{8}\int_{-4}^{0} (4+t)\, e^{-j(2\pi/8)kt}\, dt$$

- Draw $x(t)$



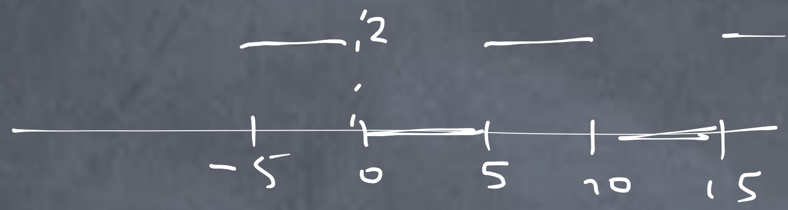
- DC? Area $\times \frac{1}{T_0} = 8 \cdot \frac{1}{8} = 1$

$$\frac{1}{8}\int_{-4}^{0} (4+t)\, dt = \frac{1}{8} \left. 4t + \frac{t^2}{2} \right|_{-4}^{0}$$

$$= 1 \,//$$

3.12 $x(t) = \begin{cases} 0 & 0 \le t \le 5 \\ 2 & 5 < t \le 10 \end{cases}$ $T_0 = 10\text{sec}$

(a) Draw $x(t)$.



(b) $a_0 = 1$ $\left(\text{Area} \times \frac{1}{10 (T_0)}\right) = \underline{1}$

(c) $a_1 = \frac{1}{10} \int_{5}^{10} 2 \cdot e^{-j(2\pi/10)t} \, dt$

$$= \frac{1}{5} \frac{e^{-j\pi/5}}{(-j\pi/5)} \Bigg|_{5}^{10} = \frac{e^{-j2\pi} - e^{-j\pi}}{-j\pi}$$

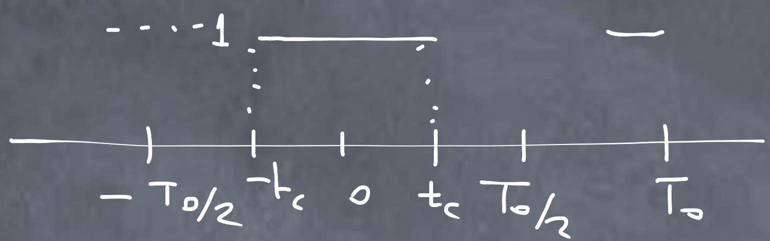
$$= \frac{1-(-1)}{-j\pi} = \frac{2j}{\pi}$$

3.9 $x(t) = x(t+T_0)$ described over $-T_0/2$ $T_0/2$

$$x(t) = \begin{cases} 1 & \text{for } |t|<t_c \\ 0 & \text{for } t_c<|t|\le \frac{T_0}{2} \end{cases}$$

$t_c < T_0/2$

(a)



(b) $$a_0 = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} x(t)\,dt = \frac{1}{T_0}\int_{-t_c}^{t_c} 1\,dt = \frac{2t_c}{t_0}$$

(c) $$a_k = \frac{1}{T_0}\int_{-t_c}^{t_c} 1\cdot e^{-jk\omega_0 t}\,dt = \frac{1}{T_0}\left.\frac{e^{-jk\omega_0 t}}{-jk\omega_0}\right|_{-t_c}^{t_c}$$

$$= \frac{1}{T_0}\,\frac{e^{-jk\omega_0 t_c} - e^{jk\omega_0 t_c}}{(-jk\omega_0)}$$

$$= \frac{2}{k\omega_0}\cdot \sin(k\omega_0 t_c)$$

Quiz $\sin 3\pi t \rightarrow$ Fourier Series

$T_0 = 2\pi/w_0$

$3\pi = 2\pi/T_0$

$\boxed{T_0 = 2/3}$

$$a_0 = \frac{3}{2}\int_0^{2/3} \sin 3\pi t\,dt = \frac{1}{3\pi}\cdot\frac{3}{2}\Big|_0^{2/3} -\cos 3\pi t$$

$$(-\cos 2\pi + \cos 0) = 0$$

$$a_k = \frac{3}{2}\cdot\frac{1}{2}\int_0^{2/3} e^{j3\pi t}\cdot e^{-j\pi/2} - e^{-j3\pi t}\cdot e^{j\pi/2}\Big) e^{-j\overset{2\pi w_0 kt}{3\pi kt}}$$

$$= \frac{3}{4}\, e^{-j\pi/2}\left(\int_0^{2/3} e^{j(3\pi t - 3\pi k t)} - e^{-j(3\pi t - 3\pi k t)}\right) dt$$

$$= \frac{3}{4}\cdot e^{-j\pi/2}\Big|_0^{2/3} (j3\pi - 3\pi k)\cdot e^{j(3\pi t - 3\pi k t)}$$

$$- \frac{3}{4}\cdot e^{-j\pi/2}\Big|_0^{2/3} (-j3\pi - 3\pi kj)\, e^{-j(3\pi t - 3\pi k t)}$$