7.8 $y[n] = \frac{1}{4}\left(x[n] + x[n-1] + x[n-2] + x[n-3]\right)$

(a) $\frac{1}{4}\left(\delta[n] + \delta[n-1] + \delta[n-2] + \delta[n-3]\right)$ Imp. Resp.

(b) System function

$H(z) = \frac{1}{4}\left(1 + z^{-1} + z^{-2} + z^{-3}\right)$ $\quad a^4 - b^4 = (a-1)(a^3 + a^2 + a + 1)$

(c) $\frac{1}{4} z^{-3}\left(z^3 + z^2 + z + 1\right) \rightarrow \frac{z^4 - 1}{z - 1}$

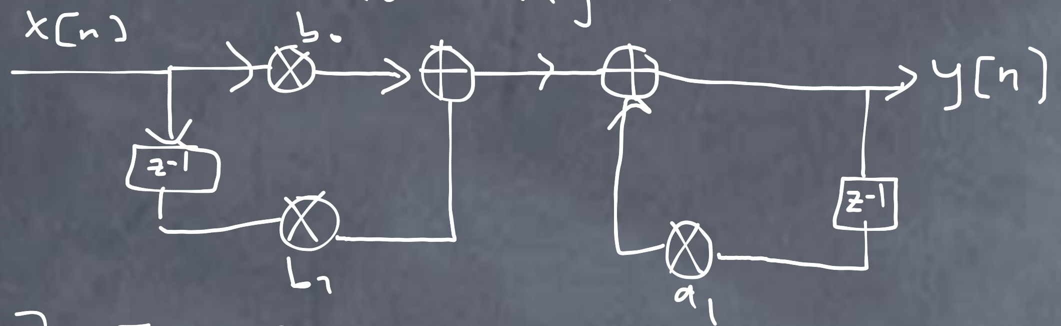
$z = -1$

$z = j$

$z = -j$

Ex 1: $y[n] = a_1 y[n-1] + b_0 x[n] + b_1 x[n-1]$

Draw block diagram.



Ex 2: Find impulse resp. of $y[n] = a_1 y[n-1] + b_0 x[n]$

$h[n] = 0$ if $n < 0$

$$h[n] = a_1 h[n-1] + b_0 \delta[n]$$

	$n<0$	0	1	2	3	4
$\delta[n]$	0	1	0	0	0	0
$h[n]$	0	b_0	$a_1 b_0$	$a_1^2 b_0$	$\cdot$	$\cdot$

$$H(z) = a_1 z^{-1} H(z) + b_0$$

$$H(z) = \frac{b_0}{1 - a_1 z^{-1}}$$

$$b_0 (a_1)^n u[n]$$

$$h[n] = b_0 (a_1)^n u[n]$$

8.1 $y[n] = \sqrt{2}\, y[n-1] - y[n-2] + x[n]$

Find impulse resp. ($h[n]=0$ $n<0$)

$$\frac{Y(z)}{X(z)} = H(z) = \frac{1}{1-\sqrt{2}z^{-1}+z^{-2}} \Rightarrow \frac{z^2}{z^2-\sqrt{2}z+1}$$

$\frac{\sqrt{2}}{2} \pm j\frac{\sqrt{2}}{2}$

$e^{\pm j\frac{\pi}{4}}$

(Pole to the n) $h[n] = A_1 (r_1)^n + A_2 (r_2)^n$

$h[n] = A_1 (e^{j\pi/4})^n + A_2 (e^{-j\pi/4})^n$

$h[0]=1$

$h[1]=\sqrt{2}$ $h[4]=-1$

$h[2]=1$

$h[3]=0$

$h[2]=1 = jA_1 - jA_2$

$h[4]=-1 = -A_1 - A_2$

$A_1 = \frac{\sqrt{2}}{2} e^{-j\pi/4}$ $A_1 = \frac{\sqrt{2}}{2} \cdot e^{j\pi/4}$

$$h[n] = \frac{\sqrt{2}}{2} e^{-j\pi/4} (e^{j\pi/4})^n + \frac{\sqrt{2}}{2} \cdot e^{j\pi/4} (e^{-j\frac{\pi}{4}})^n$$

$$= \left[\sqrt{2}\cos\left(\frac{\pi}{4}n - \frac{\pi}{4}\right)\right] u[n]$$

8.2 $y[n] = y[n-1] + y[n-2] + x[n]$

$h[0] = 1$

$h[1] = 1$

$h[2] = 2$

$h[3] = 3$

$h[4] = 5$

$$Y(z) = z^{-1}Y(z) + z^{-2}Y(z) + X(z)$$

$$(1 - z^{-1} - z^{-2})\,Y(z) = X(z)$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1}{1 - z^{-1} - z^{-2}}$$

$$\text{Poles} = \frac{1 \pm \sqrt{5}}{2}$$

$$h[n] = K_1 \left(\frac{1+\sqrt{5}}{2}\right)^n u[n] + K_2 \left(\frac{1-\sqrt{5}}{2}\right)^n u[n]$$

$$K_1 = \frac{1+\sqrt{5}}{2\sqrt{5}} \qquad K_2 = \frac{-1-\sqrt{5}}{2\sqrt{5}}$$

8.5 Determine the poles and zeros.

(a) $y[n] = \frac{1}{2}y[n-1] - \frac{1}{3}y[n-2] - x[n]$

$$Y(z) = \frac{1}{2}z^{-1}Y(z) - \frac{1}{3}z^{-2}Y(z) - X(z)$$

$$\left(1 - \frac{1}{2}z^{-1} + \frac{1}{3}z^{-2}\right)Y(z) = -X(z)$$

$$H(z) = \frac{-1}{1-\frac{1}{2}z^{-1}+\frac{1}{3}z^{-2}} = \frac{-z^2}{z^2-\frac{1}{2}z+\frac{1}{3}}$$

$$z = 0.25 \pm j0.52$$

$$z = 0.57\, e^{\pm j0.357\pi}$$

(Poles)

8.7 $y[n] = -0.8y[n-1] + 0.8x[n] + x[n-1]$

(a) $Y(z) = -0.8z^{-1}Y(z) + 0.8X(z) + z^{-1}X(z)$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{0.8 + z^{-1}}{1 + 0.8z^{-1}} = \frac{0.8z + 1}{z + 0.8}$$

Pole at $z = -0.8$

Zero $= z = -1.25$

(c) $$H(e^{j\omega}) = H(z)\Big|_{z = e^{j\omega}} = \frac{0.8 + e^{-j\omega}}{1 + 0.8e^{-j\omega}}$$

8.9 $y[n] = -0.9\,y[n-6] + x[n]$

$$Y(z) = -0.9 z^{-6} Y(z) + X(z)$$

$$H(z) = \frac{1}{1+0.9z^{-6}} = \frac{z^6}{z^6+0.9}$$

Poles $z^6 = -0.9 = 0.9\, e^{j\pi} \cdot e^{j2\pi l}$

$$= \sqrt[6]{0.9}\; e^{j\pi/6}\; e^{j\pi l/3}$$

$$= \underbrace{0.98}\; e^{j\pi(2l+1)/6}$$

$\pi/6,\ 3\pi/6,\ 5\pi/6 \ldots$

8.10 $y[n] = -\frac{1}{2} y[n-1] + x[n]$

$Y(z) = -\frac{1}{2} z^{-1} Y(z) + X(z)$

$(1 + \frac{1}{2} z^{-1}) Y(z) = X(z)$

$H(z) = \frac{1}{1 + \frac{1}{2} z^{-1}}$ $\qquad z = -\frac{1}{2}$ (pole)

(b) $H(z) = \frac{1}{1 + \frac{1}{2} z^{-1}} \rightarrow h[n] = \left(-\frac{1}{2}\right)^n u[n]$

when $x[n] = \delta[n] + \delta[n-1] + \delta[n-2]$

$y[n] = h[n] + h[n-1] + h[n-2]$

$y[n] = \left(-\frac{1}{2}\right)^n u[n] + \left(-\frac{1}{2}\right)^{n-1} u[n-1] + \left(-\frac{1}{2}\right)^{n-2} u[n-2]$

Example: (Inv z transform) Partial fraction

$$X(z) = \frac{1-2.1z^{-1}}{1-0.3z^{-1}-0.4z^{-2}}$$

$$= \frac{1-2.1z^{-1}}{(1+0.5z^{-1})(1-0.8z^{-1})}$$

$$X(z) = \frac{A}{1+0.5z^{-1}} + \frac{B}{1-0.8z^{-1}}$$

$$A \to X(z) \quad (1+0.5z^{-1}) \Big|_{z=-0.5}$$

$$\left.\frac{1-2.1z^{-1}}{1-0.8z^{-1}}\right|_{z=-0.5} = \frac{1+4.2}{1+1.6} = 2$$

$$B \to X(z) \quad (1-0.8z^{-1}) \Big|_{z=0.8}$$

$$\left.\frac{1-2.1z^{-1}}{1+0.5z^{-1}}\right|_{z=0.8} = \frac{1-2.1/0.8}{1+0.5/0.8} = -1$$

$$X(z) = \frac{2}{1+0.5z^{-1}} - \frac{1}{1-0.8z^{-1}}$$

$$x[n] = 2(-0.5)^n u[n] - (0.8)^n u[n]$$

$$H(z) = \frac{b_0}{1-a_1 z^{-1}} \Rightarrow h[n] = b_0 \cdot a_1^n u[n] \quad (\text{Pg. } 217)$$

Long Division: Ex:

$$Y(z) = \frac{2 - 2.4z^{-1} - 0.4z^{-2}}{1 - 0.3z^{-1} - 0.4z^{-2}}$$

$$\begin{array}{rrrl} & -0.4z^{-2} & -2.4z^{-1} & +2 \\ - & -0.4z^{-2} & -0.3z^{-1} & +1 \quad (\times 1) \\ \hline & & -2.1z^{-1} & +1 \end{array}$$

$$Y(z) = \frac{1 - 2.1z^{-1}}{(1+0.5z^{-1})(1-0.8z^{-1})} + 1$$

$$y[n] = 2 \cdot (-0.5)^n u[n] - (0.8)^n u[n] + \delta[n]$$

8.11 (a) $\dfrac{1 - z^{-1}}{1 + 0.77 z^{-1}}$

$$\dfrac{1 - z^{-1}}{1 + 0.77 z^{-1}} \Big/ \dfrac{1}{0.77}$$

$$\dfrac{1}{0.77} + 1$$

$$LD = +1.3 + \frac{2.3}{1 + 0.77 z^{-1}}$$

$$h[n] = +1.3\,\delta[n] + 2.3\,(-0.77)^n\,u[n]$$

(c) $H(z) = \dfrac{z^{-2}}{1 - 0.9 z^{-1}} = z^{-2}\left(\dfrac{1}{1 - 0.9 z^{-1}}\right)$

$$h[n] = (0.9)^{n-2}\,u[n-2]$$

$G(z)$

$$g[n] = (0.9)^n\,u[n]$$

$$g[n-2] = (0.9)^{n-2}\,u[n-2]$$