5.8 $b_k = \{13, -13, 13\}$

$x[n] = \begin{cases} 0 & \text{for } n \text{ even} \\ 1 & \text{for } n \text{ odd} \end{cases}$

$y[n] = ?$

$h[n] = 13\delta[n] - 13\delta[n-1] + 13\delta[n-2]$

	13	0	13	0	13	0	13	0	13
	0	-13	0	-13	0	-13	0	-13	0
+	13	0	13	0	13	0	13	0	13
	26	-13	26	-13	26	-13	...		

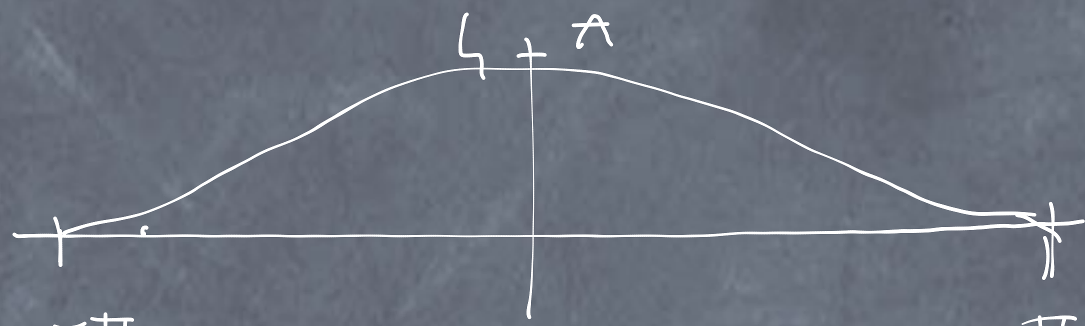
$y[n] = \begin{cases} -13 & \text{for } n \text{ even} \\ 26 & \text{for } n \text{ odd} \end{cases}$

Ex: $b_k = \{1, 2, 1\}$ $\quad y[n] = x[n] + 2x[n-1] + x[n-2]$

$$H(e^{j\omega}) = 1 + 2\cdot e^{-j\omega} + e^{-j\omega 2}$$

$$= e^{-j\omega}\left(e^{j\omega} + 2 + e^{-j\omega}\right)$$

$$= \underbrace{e^{-j\omega}}_{\text{phase}}\underbrace{(2 + 2\cos\omega)}_{\text{magnitude}}$$



Put $\quad x[n] = 2\,e^{j\pi/4}\cdot e^{j\pi n/3}$

$$H(e^{j\omega}) = H(e^{j\cdot\pi/3}) = ?$$

$$= 3\cdot e^{-j\pi/3}$$

$$x[n] = \underbrace{\text{comp}}_{\omega_1}\cdot H(e^{j\omega_1}) + \underbrace{\text{comp}}_{\omega_2} H(e^{j\omega_2})$$

$$y[n] = 2\cdot e^{j\pi/4}\cdot e^{j\pi n/3}\cdot 3\cdot e^{-j\pi/3}$$

6.7 $y[n] = 2x[n] - 3x[n-1] + 2x[n-2]$

a. Find freq resp. $H(e^{jw})$

$$\underset{\overset{\uparrow}{w}}{H(e^{jw})} = 2 - 3 \cdot e^{-jw} + 2 \cdot e^{-j2w}$$

$$H(e^{jw}) = e^{-jw}\left(2 \cdot e^{jw} + 2 \cdot e^{-jw} - 3\right)$$

$$= \underbrace{e^{-jw}} \; \underbrace{(4 \cdot \cos w - 3)}$$

d. Find all frequencies $\hat{w}$ output response is zero.

$$4\cos w = 3$$

$$\cos w = \frac{3}{4} \qquad w \Rightarrow \pm 0.23\pi$$

6.5 $y[n] = x[n] + 2x[n-1] + x[n-2]$

(c) Determine the output when input

$$x[n] = 10 + 4\cos(0.5\pi n + \pi/4)$$

$$H(\omega) = (2 + 2\cos\omega)\, e^{-j\omega}$$

$H(0) = 4$ $\quad H(\pi/2) = 2 \cdot e^{-j\pi/2}$ $\quad H(-\frac{\pi}{2}) = 2 \cdot e^{j\pi/2}$

$$y[n] = 4 \times 10 + H(\pi/2) \cdot 2 \cdot e^{j\pi/4}\, e^{j\pi/2 n} + H(-\tfrac{\pi}{2}) \cdot e^{-j\pi/4} \cdot e^{-j\frac{\pi}{2}n}$$

$$= 40 + 8\cos\left(\frac{\pi}{2} n - \frac{\pi}{4}\right)$$

6.6 $y[n] = x[n] - x[n-2]$

(a) $H(\omega) = 1 - e^{-j2\omega}$

freq. resp.

$= e^{-j\omega}\left(e^{j\omega} - e^{-j\omega}\right)$

$= e^{-j\omega} \cdot 2 \cdot e^{j\pi/2} \cdot \sin\omega$

(c) $x[n] = 4 + \cos\left(\frac{\pi}{4}n - \frac{\pi}{4}\right)$

$H(0)$ $\qquad H\left(\frac{\pi}{4}\right)$

$H(0) = 0$

$H\left(\frac{\pi}{4}\right) = \sqrt{2}\cdot e^{j\pi/4}$

$H\left(-\frac{\pi}{4}\right) = -\sqrt{2}\cdot e^{j3\pi/4}$

$4 \cdot 0 + \sqrt{2}\cos\left(\frac{\pi}{4}n\right)$

A

$-\frac{\pi}{4} + \frac{\pi}{4}$

$4 + \frac{1}{2}\cdot e^{j\frac{\pi}{4}n} \cdot e^{-j\frac{\pi}{4}} + \frac{1}{2}\cdot e^{-j\frac{\pi}{4}n}\cdot e^{-j\frac{\pi}{4}}$

$= 4 \cdot 0 + \frac{1}{2}\cdot\sqrt{2}\cdot e^{j\pi/4} \cdot e^{j\frac{\pi}{4}n}\cdot e^{-j\frac{\pi}{4}} - \frac{\sqrt{2}}{2}\cdot e^{-j\frac{\pi}{4}n}\cdot e^{j\pi}$

$= \frac{\sqrt{2}}{2} e^{j\frac{\pi}{4}n} + \frac{\sqrt{2}}{2}\cdot e^{-j\frac{\pi}{4}n}$

$= \sqrt{2}\cos\left(\frac{\pi}{4}n\right)$

6.7 $H(e^{j\omega}) = 1 + 2 \cdot e^{-j3\omega}$

$h[n] = \delta[n] + 2\delta[n-3]$

(b) $H(e^{j\omega}) = 2 \cdot e^{-j3\omega} \cos\omega$

$= e^{-j3\omega}(e^{j\omega} + e^{-j\omega})$

$= e^{-j2\omega} + e^{-j4\omega}$

$h[n] = \delta[n-2] + \delta[n-4]$

6.8 $H(e^{j\omega}) = (1+e^{-j\omega})(1-e^{j\frac{2\pi}{3}} \cdot e^{-j\omega})(1-e^{-j\frac{2\pi}{3}} e^{-j\omega})$

a. Diffence Eqn.

$= (1+e^{-j\omega})(1-e^{-j\frac{2\pi}{3}} \cdot e^{-j\omega} - e^{j\frac{2\pi}{3}} e^{-j\omega} + e^{-j2\omega})$

$= (1+e^{-j\omega})(1-e^{-j\omega} + e^{-j2\omega})$

$= 1 + e^{-j3\omega}$ $\qquad y[n] = x[n] + x[n-3]$

b. Imp. Resp. $h[n] = \delta[n] - \delta[n-3]$

7.1 $x_1[n] = \delta[n]$

$X_1(z) = 1$

b) $x_2[n] = \delta[n-1]$

$X_2(z) = z^{-1} \cdot X_1(z)$

time delay property

$= z^{-1}$

c) $x_3[n] = \delta[n-7]$

$X_3(z) = z^{-7} \cdot X_1(z)$

$= z^{-7}$

$x[n] = \sum_{k=0}^{\infty} x[k]\delta[n-k]$

$X(z) = \sum x[k] z^{-k}$

Reminder

d.

$x_4[n] = 2\delta[n] - 3\delta[n-1] + 4\delta[n-3]$

$= 2 - 3 \cdot z^{-1} + 4 \cdot z^{-3}$

7.2 $H(z) = ?$

$$y[n] = x[n] - x[n-1]$$
$$Y(z) = X(z) - X(z) \cdot z^{-1}$$
$$Y(z) = X(z)(1 - z^{-1})$$

$$\underline{y[n] = x[n] * h[n]}$$
$$Y(z) = X(z) \cdot H(z)$$

$$\frac{Y(z)}{X(z)} = H(z)$$

$$H(z) = 1 - z^{-1}$$

7-3 $H(z) = 1 + 5z^{-1} - 3z^{-2} + 2.5z^{-3} + 4z^{-8}$

ⓐ Find difference eqn.

$y[n] = x[n] + 5x[n-1] - 3x[n-2] + 2.5x[n-3] + 4x[n-8]$

ⓑ when input $x[n] = \delta[n]$ determine output.

$h[n] = \delta[n] + 5\delta[n-1] - 3\delta[n-2] + 2.5\delta[n-3] + 4\delta[n-8]$

actually (find $h[n]$)

7.4 $\quad y[n] = \frac{1}{3}\left(x[n] + x[n-1] + x[n-2]\right)$

ⓐ Determine $H(z) \rightarrow$ system function

$$H(z) = \frac{1}{3} + \frac{1}{3}z^{-1} + \frac{1}{3}z^{-2}$$

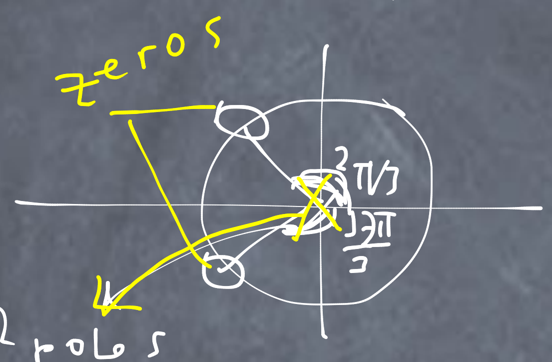
ⓑ Plot poles & zeros

$$\frac{1}{z^2}\left(\frac{1}{3}\cdot z^2 + \frac{1}{3}z + \frac{1}{3}\right) \rightarrow z = \frac{-1 \pm \sqrt{1-4}}{2}$$

$\frac{1}{z^2}$ ↳ root 0, 0 → pole

$$= -\frac{1}{2} \pm j\frac{\sqrt{3}}{2}$$

$$= 1\, e^{\pm j\frac{2\pi}{3}}$$



ⓒ $H(\omega) = H(e^{j\omega}) = H(z)\Big|_{z = e^{j\omega}}$

$$= \frac{1}{3} + \frac{1}{3}e^{-j\omega} + \frac{1}{3}\cdot e^{-j2\omega} = \frac{1}{3}\cdot e^{-j\omega}\left(e^{j\omega} + 1 + e^{-j\omega}\right)$$

$$= \frac{1}{3}\cdot e^{-j\omega}\left(e^{j\omega} + 1 + e^{-j\omega}\right)$$

$$H(\omega) = \underbrace{e^{-j\omega}}_{\text{phase}}\underbrace{\left(\frac{2\cos\omega + 1}{3}\right)}_{\text{mag.}}$$

ⓓ $x[n] = 4 + \cos(0.25\pi(n-1)) - 3\cos(2\pi/3 \cdot n)$

$$y[n] = 4\cdot H(0) + |H(\pi/4)|\cos\left(0.25\pi n - 0.25\pi + \angle H(\pi/4)\right)$$

$$-3\,|H(2\pi/3)|\cos\left(2\pi/3\cdot n + \underbrace{\angle H(2\pi/3)}_{\text{phase}}\right)$$

$$= 4 + \left(\frac{\sqrt{2}+1}{3}\right)\cdot\cos\left(\frac{\pi n}{4} - \frac{\pi}{4} - \frac{\pi}{4}\right) - 3\cdot 0 \ldots$$

$$= 4 + \left(\frac{\sqrt{2}+1}{3}\right)\cdot\cos\left(\frac{\pi n}{4} - \frac{\pi}{2}\right)$$