CMPE 300 ANALYSIS OF ALGORITHMS 29.3.2016
MIDTERM

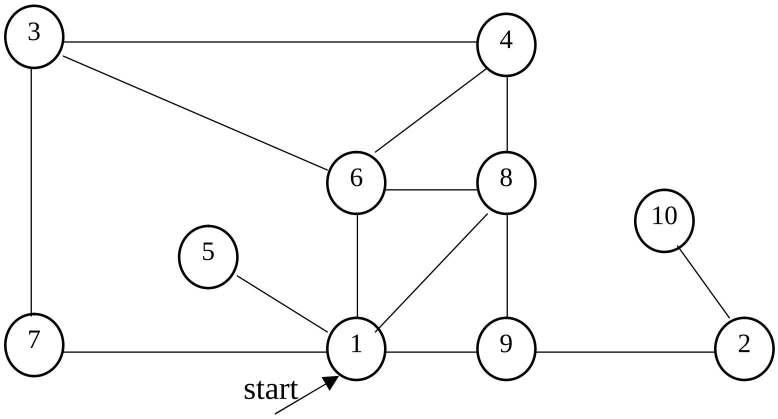
1. Consider the recurrence relation below:

$$T(n) = \left\lfloor \sum_{i=1}^{n-1} T(i)T(i-1) + 1 \right\rfloor + T(n-1), \qquad T(0) = T(1) = 1, n > 1$$

   a) Write a recursive algorithm (in pseudocode)that computes T(n) directly as given above. Show that the algorithm performs an exponential number of operations (i.e. solve the above recurrence relation).
   b) It is possible to compute T(n) in a more efficient way, without recalculating the same T(i) values twice as shown above. Write a more efficient algorithm (in pseudocode) that computes T(n), which performs $\Theta(n^2)$ operations. Show that it is $\Theta(n^2)$. (Your algorithm must be $\Theta(n^2)$.)
   c) Write an even more efficient algorithm (in pseudocode) that computes T(n), which performs $\Theta(n)$ operations.Show that its is $\Theta(n)$.

2. We know that insertion sort is an efficient algoithm for lists that are nearly sorted. Give a theorem related to this property of insertion sort (i.e. describe this property *formally*). Then prove the theorem.

3. Consider the graph below:



   a) Apply depth-first search. Show clearly how the nodes are visited and explain in detail. List the order of node visits.
   b) Apply breadth-first search. Show clearly how the nodes are visited and explain in detail.List the order of node visits.

4. We are given the following algorithm:

```
function Module (L[low:high])
      if (high > low) then
          for i=low+1 to high do
              print (...)
              if (L[i] > L[low]) then
                  print (...)
              endif
          endfor
          temp = random (low..high)
          call Module (L[low:temp])
          call Module (L[temp:high])
      endif
end
```

(continued on next page)

The basic operation is the "print(...)" statements. The algorithm is initially called as "call Module (L[1:n])". Assume that each element of the array L is a distinct integer between 1 and n. The "random(a..b)" function returns a random number (integer) between integers *a* and *b* (inclusive), a≥b. Calculate A(n)*exactly*.

Some equations that you may use:

- $\sum_{i=0}^{n-1} x^i = \frac{x^n - 1}{x - 1}$, for x≠1
- For a binary tree T with L leaf nodes, $LPL(T) \geq L\lfloor \log_2 L \rfloor + 2\left(L - 2^{\lfloor \log_2 L \rfloor}\right)$, where LPL(T) is the leaf path length of T.

Notes:

- Where pseudocode is required, the syntax of the pseudocode must be strictly followed. No points will be given if the syntax is not followed or any other language (e.g. C) is used.
- Questions 1,4:30 points; 2,3:20 points
- Time: 2:00 hours
- Close notes and books

Herkesin bir feride'si vardır bilmez miyim
Herkesin bir ayakkabısı gibi bir de şarkısı
Herkesin bir kimsesi vardır ben bilmez miyim
Bir de kimsesizliği...