

CMPE 300 - Analysis of Algorithms
Fall 2016
Assignment 1

Due: March 22, 17:00

This is an individual homework. Please answer the following questions and submit your answers to your assistant both as hard copy and soft copy. The sole purpose of this homework is to familiarize you with the processes involved in answering complexity related questions. Please work on them by your own. Please do not submit just an answer, but show all your reasoning, and how you arrive at the answers. For any further questions, contact the assistant.

Question 1

(40) For this problem, you should use the definition $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$

1. Show that $e^x \geq 1 + x$ for all real $x \geq 0$ and also that $e^k \geq \frac{k^k}{k!}$ for $k \in \mathbb{Z}^+$.
2. Prove that $\binom{n}{k} \geq \left(\frac{n}{k}\right)^k$ for integers $0 < k < n$.
3. Prove that $\binom{n}{k} \leq \left(\frac{en}{k}\right)^k$ for integers $0 < k < n$.
4. Prove that $k \ln \frac{n}{k} \leq \ln \binom{n}{k} \leq k \ln \frac{n}{k} + k$ for integers $0 < k < n$.

Question 2

(60) Do a fine (exact) analysis and calculate the number of operations for the worst case and average case. Express the worst case and average case time complexities of the function using the big O notation. The basic operations are assignments and condition checks, which each consume 1 unit of time. You can ignore the assignments in the loop headers.

Function: $(X[0 : n - 1], k)$

Input: $X[0 : n - 1]$ array of elements where $X[i] \in \{0, 1\}$,
 k a nonnegative integer less than or equal to n

Output: A number

```
    for  $i \leftarrow 0$  to  $n - 1$  do
        for  $j \leftarrow 0$  to  $i$  do
             $a \leftarrow a + X[j]$ 
        end for
         $A[i] \leftarrow a / (i + 1)$ 
    end for
    for  $i \leftarrow 0$  to  $n - 1$  do
         $b \leftarrow b + X[i]$ 
    end for
    if  $b = k$  then
        for each subset  $S$  of  $\{0, 1, \dots, n - 1\}$  do
            for  $j \leftarrow 1$  to  $k^k$  do
                 $d \leftarrow d + 1$ 
            end for
        end for
        return  $d$ 
    end if
    else
        return  $b$ 
    end if
end
```